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ON THE MODULE STRUCTURE IN A CYCLIC EXTENSION OVER A $\mathfrak{p}$-ADIC NUMBER FIELD

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Let p be a prime. Let k be a $\mathfrak{p}$-adic number field and $\mathfrak{o}$ be the ring of all integers of k. Let K/k be a cyclic totally ramified extension of degree p^n with Galois group G. Clealy the ring $\mathfrak{O}$ of all integers of K is an $\mathfrak{o}[G]$-module, and the purpose of this paper is to give a necessary and sufficient condition for the $\mathfrak{o}[G]$-module $\mathfrak{O}$ to be indecomposable.

In §§ 1–2, we shall prepare some lemmas. In §§ 3–4, we shall obtain the necessary and sufficient condition (Theorem 3).

Throughout this paper, let π be a prime element of k and e be the absolute ramification index of k. For a positive rational integer a, we define a function $m(a)$ by

$$m(a) = \left[\frac{(p-1)(a+1)}{p}\right] .$$

1.

In this section, we shall obtain some inequalities for ramification numbers. Let F/k be a cyclic ramified extension of prime degree p with the first ramification number b. Let $\mathfrak{O}_F$ be the ring of all integers of F. Let e, π and $m(a)$ be the same as in Introduction. Then it is well known that

$$m(b) \leqq e$$

and

$$\mathrm{tr}_{F/k}\, \mathfrak{O}_F = (\pi^{m(b)}) , \tag{1}$$

where $\mathrm{tr}_{F/k}$ denotes the trace map from F to k (for example, see [2]).

Let ζ be a primitive p-th root of 1. Let F' and k' be the extensions

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