L. Bădescu
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A REMARK ON THE GROTHENDIECK-LEFSCHETZ THEOREM ABOUT THE PICARD GROUP

LUCIAN BĂDESCU

Let K be an algebraically closed field of arbitrary characteristic. The term "variety" always means here an irreducible algebraic variety over K. The notations and the terminology are borrowed in general from EGA [4].

Let X be a projective non-singular variety embedded in the projective space $P^n = P$, and let Y be a closed subvariety. Throughout this note we shall assume that $\dim(Y) \geqslant 2$ and that Y is a scheme-theoretic complete intersection of X with some hypersurfaces $H_1, \cdots, H_r$ of P, where $r = \mathrm{codim}_X(Y)$. Sometimes we shall simply say that Y is complete intersection in X.

First of all recall the following result (see [5], [7]):

THEOREM A (Grothendieck-Lefschetz). *In the above hypotheses, assume moreover that K is the complex field and that Y is non-singular of dimension $\geqslant 3$. Then the natural homomorphism of restriction of Picard groups*

$$\mathrm{Pic}(X) \to \mathrm{Pic}(Y) \tag{1}$$

is an isomorphism.

Note. There is in fact a more precise statement than the above theorem, asserting that even the corresponding morphism between Picard schemes $\underline{\mathrm{Pic}}(X) \to \underline{\mathrm{Pic}}(Y)$ is an isomorphism.

The above theorem implies in particular that the following homomorphisms of restriction are also isomorphisms (the hypotheses being the same as in theorem A):

$$\mathrm{Pic}(X)/Z[O_X(1)] \to \mathrm{Pic}(Y)/Z[O_Y(1)] \tag{2}$$

$$\mathrm{Pic}^\tau(X) \to \mathrm{Pic}^\tau(Y) \tag{3}$$

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