ON GLOBAL CLUSTER SETS FOR FUNCTIONS
MEROMORPHIC ON SOME RIEMANN SURFACES

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0. Consider a single-valued meromorphic function \( w = f(p) \) defined on an open Riemann surface \( R \) with an ideal boundary \( \beta \). In [1], Collingwood and Cartwright introduced the global cluster set for a function meromorphic on the unit disk. Generalizing the definition of global cluster sets to our present setting, we define the global cluster set for \( w = f(p) \) as follows:

A value \( w \) in the extended complex plane is called a cluster value at \( \beta \) if there exists a sequence \( \{ p_n \}_{n=1}^\infty \) in \( R \) converging to \( \beta \) such that

\[
\lim_{n \to \infty} f(p_n) = w .
\]

The set consisting of cluster values is called the global cluster set for \( w = f(p) \) and denoted by \( C_R(f) \).

In the same way, the range of values \( R_R(f) \) and the asymptotic set \( A_R(f) \) can be defined as usual in our present setting. Collingwood and Cartwright obtained the following so-called their main theorem in their setting:

**THEOREM.** \( C_R(f) \) is defined as \( \{ \text{Int } R_R(f) \} \cup \overline{A_R(f)} \), where \( \text{Int} \) and the bar indicate the interior and the closure, respectively.

Using the wholly analogous discussion to their proof, we can prove, although we omit the proof, that the above theorem is valid for our present setting.

By the reason that the realization of ideal boundaries in our definition of global cluster sets is extremely rough, one might doubt that any refined function-theoretic information can be derived from global cluster sets. However, not only global cluster sets are convenient to

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