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QUANTUM WHITE NOISES

WHITE NOISE APPROACH TO QUANTUM STOCHASTIC CALCULUS*

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I. Introduction

Let $H = L^2(R)$ be the Hilbert space of all complex-valued square integrable functions defined on R, $\Phi = \Gamma(H)$ be the Boson Fock space over H. For each $h \in H$, denote by $\varepsilon(h)$ the corresponding exponential vector:

$$\varepsilon(h) = 1 \oplus h \oplus h^{\otimes 2}/2! \oplus \cdots h^{\otimes n}/n! \oplus \cdots,$$

in particular $\varepsilon(0)$ is the Fock vacuum. It is well known (cf. [9,15]) that the family $\mathbf{E} = \{\varepsilon(h) ; h \in H\}$ is linearly independent and total in Φ. In developing their quantum stochastic calculus, Hudson and Parthasarathy [9] used the set $\mathbf{E}$ as "testing vectors": all operators on Φ were firstly defined on $\mathbf{E}$ and then extended to their proper domains. Instead of operator valued processes, they essentially dealt with vector valued ones and, therefore, obtained a quantum (i.e. noncommutative) version of Itô's product formula which was only based on the commutation rules of a free Boson field and Lebesgue integration. The three fundamental integrators are annihilation, creation and number processes which played the role of "quantum noises" in quantum stochastic evolutions. They are noncommutative extensions of classical Brownian motion and Poisson process.

On the other hand, the white noise approach initiated by T. Hida [5] has been proved highly effective to the classical stochastic integration theory. One natural question is: what one can do with it in quantum stochastic calculus?

In the present paper, we define the "quantum white noise" as a generalized quantum process in terms of Hida's derivative (or "causal calculus"). Since it could be rigorously treated in the framework of Hida's distributions over white noise space rather than in Fock space Φ, we establish some kind of chaos decomposition for operators which is a considerable extension of those decompositions for

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