WHITE NOISE DELTA FUNCTIONS AND
CONTINUOUS VERSION THEOREM

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Introduction

The recently developed Hida calculus of white noise [5] is an infinite dimen-
sional analogue of Schwartz’ distribution theory based on the Gelfand triple
\((E) \subset (L^2) = L^2(E^*, \mu) \subset (E)^*\), where \((E^*, \mu)\) is Gaussian space and \((L^2)\) is
(a realization of) Fock space. It has been so far discussed aiming at an application
to quantum physics, for instance [1], [3], and infinite dimensional harmonic analy-
sis [7], [8], [13], [14], [15]. During the development an important milestone was
Kubo-Yokoi’s continuous version theorem [11] which asserts that every test white
noise functional \(\phi \in (E)\) admits a unique continuous version and, therefore, the
test functionals constitute a space of continuous functions on \(E^*\). This theorem is
very fundamental and indispensable for many arguments. For example, it allows
us to introduce a delta function on Gaussian space, which is one of the most
important generalized functions. Furthermore, the continuous version theorem is
effectively applied to description of positive generalized white noise functionals
[19].

The motivation of this paper is to give an alternative proof of the continuous
version theorem by means of a direct use of defining Hilbertian seminorms of \(E^*\).
In fact, this approach yields a sharp estimate of white noise delta functions
\(\delta_x \in (E)^*, x \in E^*\), from which the continuous version theorem follows. Moreover,
with this method we may prove the continuity of \(x \mapsto \delta_x \in (E)^*, x \in E^*\), which
guarantees that the \(n\)-fold (topological) tensor product \((E) \otimes \cdots \otimes (E)\) is again
a space of continuous functions on the product of the Gaussian space \(E^* \times \cdots \times
E^* (n \text{ times})\).

Here we remark some closely related works. In [12] Lee proved that each test
functional \(\phi \in (E)\) admits an analytic version on each Hilbert space \(E_{-p}\), where
\(E^* = \text{ind lim}_{p \to \infty} E_{-p}\). However, since the inductive system \((E_{-p})_{p \geq 0}\) is not strict,
our continuous version theorem does not follow from his result. In [9] Kondrat’ev

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