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Nagoya Math. J.
Vol. 128 (1992), 1–14

ON A BINGHAM FLUID WHOSE VISCOSITY AND YIELD LIMIT DEPEND ON THE TEMPERATURE

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Introduction

Duvaut and Lions [2] studied the field of velocities and of temperatures in a moving incompressible Bingham fluid endowed with viscosity $\mu(\theta)$ depending on the temperature θ and established the existence of a weak solution in the case of a two dimensional fluid. However, the problem of uniqueness remained unsolved. The purpose of the present paper is to give an affirmative answer to the problem, that is, to show the local existence (resp. the global existence) in the time and the uniqueness of (strong) solutions in three dimensions under the conditions that (i) the time (resp. the initial velocity and the external force) and (ii) the rate of variation of the viscosity and the yield limit with respect to the temperature are both sufficiently small. It will be easily seen that the global existence and the uniqueness also hold in the two dimensional case whenever the rate (ii) is sufficiently small.

The general plan of the proof follows the analogous lines as in [2]. Let ϕ be a given function. We first find the unique velocity field u_ϕ of a Bingham fluid with viscosity $\mu(\phi)$ and yield limit $g(\phi)$, employing Theorem 3 of Kato [4], and secondly seek the solution θ_ϕ of the heat equation $\theta_t - \Delta\theta = G_\phi$, the equation of energy-conservation associated with u_ϕ, with the aid of the theorem due to Grisvard [3]. A desired field of temperature is obtained by a fixed point θ of the mapping $H : \phi \to \theta_\phi$ and u_θ is a desired field of velocity. The crucial point will be in finding an auxiliary Banach space X to which ϕ belongs and on which mapping H becomes compact, and in estimating the right hand side G_ϕ of the heat equation in terms of $\| \phi \|_X$ (see Lemma 2.2) so that a ball in X is transformed into itself by mapping H under some circumstances.

The main result, Theorem 1, is described in Section 1. The aim of Section 2 is to get u_ϕ and θ_ϕ. Section 3 is devoted to the proof of Theorem 1 in which

Received July 22, 1991.