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MUKAI-UMEMURA'S EXAMPLE OF THE FANO THREEFOLD WITH GENUS 12 AS A COMPACTIFICATION OF C^3

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§0. Introduction

Let (X, Y) be a smooth projective compactification with the non-normal irreducible boundary Y, namely, X is a smooth projective algebraic threefold and Y a non-normal irreducible divisor on X such that $X - Y$ is isomorphic to C^3. Then Y is ample and the canonical divisor K_X on X can be written as $K_X = -rY (1 \leqq r \leqq 4)$. Thus X is a Fano threefold. In particular, $\mathrm{Pic}\, X \cong \mathbf{Z}\mathcal{O}_X(Y)$. The non-normality of Y implies that $r \leqq 2$ (cf. [4]). In the case of $r = 2$, such a (X, Y) is uniquely determined up to isomorphism, in fact, $(X, Y) \cong (V_5, H_5^\infty)$, where $X = V_5$ is a Fano threefold of degree 5 in $\mathbf{P}^6$, and $Y = H_5^\infty$ is a ruled surface swept out by lines which intersect the line Σ with the normal bundle $N_{\Sigma|X} \cong \mathcal{O}_\Sigma(-1) \oplus \mathcal{O}_\Sigma(1)$, in particular, Σ is the singular locus of Y. In the case of $r = 1$, there is an example of such a compactification of $\mathbf{C}^3$, in fact, let $X = V_{22}'$ be a Fano threefold of genus $g = 12$ constructed by Mukai-Umemura [11] and $Y = H_{22}'$ be the ruled surface swept out by conics which intersect the line ℓ in V_{22}' with the normal bundle $N_{\ell|X} \cong \mathcal{O}_\ell(-2) \oplus \mathcal{O}_\ell(1)$, then H_{22}' is a non-normal hyperplane section of V_{22}' such that $V_{22}' - H_{22}'$ is isomorphic to $\mathbf{C}^3$, in particular, the line ℓ is the singular locus of H_{22}' (cf. [6]).

Now, in this paper, we will construct a birational map $\pi : V_{22}' \cdots\to V_5$ such that the restriction π_0 of π on $V_{22}' - H_{22}'$ gives an isomorphism $V_{22}' - H_{22}' \cong V_5 - H_5^\infty \cong \mathbf{C}^3$, via the resolution of indeterminancy of the double projection of V_{22}' from the singular locus $\mathrm{Sing}\, H_{22}'$ of H_{22}' which is a line on V_{22}' (see Theorem 1). Furthermore, we will study the detailed structure of the desingularization and the normalization of the boundary divisor H_{22}' (see Theorem 2).

Recently, Mukai ([11_a]) proved that there is a 4-dimensional family of Fano threefolds of first kind with index one, genus 12 which are the compactifications of $\mathbf{C}^3$ with non-normal boundaries, in particular, our example (V_{22}', H_{22}') belongs

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