J. Satoh
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A CONSTRUCTION OF q-ANALOGUE OF DEDEKIND SUMS

JUNYA SATOH

0. Introduction

If one looks back the classical proof (cf. Carlitz [4]) of the reciprocity law for Dedekind sums in order to construct q-analogue of Dedekind sums which also have the reciprocity law, one can soon see that the following elementary equation is essential in the proof:

$$(1) \qquad \frac{1-u}{e^t-u}\frac{1-v}{e^t-v} = \frac{1-v}{u-v}\frac{1-u}{e^t-u} + \frac{1-u}{v-u}\frac{1-v}{e^t-v},$$

for any distinct complex numbers u and v, where $\frac{1-u}{e^t-u}$ means the generating function of Euler numbers associated to u. So we must extend the above equation to the generating function of q-Euler numbers for our purpose. As a result, we obtain a very suggestive equation (see Lamma 5) under the conditions $|u|>1$ and $|v|>1$:

$$(2) \qquad F_{u;q}(t) * F_{v;q}(t) = \frac{1-v}{u-v}F_{u;q}(t) + \frac{1-u}{v-u}F_{v;q}(t),$$

where $F_{u;q}(t)$ means the generating function of q-Euler numbers associated to u and the left hand side of (2) is determined by Lemma 4. The above equation is correspond to the decomposition into partial fractions of $\frac{1-u}{e^t-u}\frac{1-v}{e^t-v}$: (1). We take a deep interest in the invariance of the form. By the generalization of the theory, we give a new method of construction of q-analogue of formal power series. In the following, we explain about the essence of our theory. In [2], Carlitz defined q-Bernoulli numbers for a complex number q as follows:

$$\beta_0(q) = 1, \quad q(q\beta(q)+1)^n - \beta_n(q) = \begin{cases} 1 & \text{for } n = 1, \\ 0 & \text{for } n > 1, \end{cases}$$

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