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ON THE LEAST DEGREE OF POLYNOMIALS BOUNDING ABOVE THE DIFFERENCES BETWEEN LENGTHS AND MULTIPLICITIES OF CERTAIN SYSTEMS OF PARAMETERS IN LOCAL RINGS

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§ 1. Introduction

Let A be a commutative local Noetherian ring with the maximal ideal $\mathfrak{m}$ and M a finitely generated A-module, $d = \dim M$. It is well-known that the difference between the length and the multiplicity of a parameter ideal $\mathfrak{q}$ of M

$$I_M(\mathfrak{q}) = l(M/\mathfrak{q}M) - e(\mathfrak{q}; M)$$

gives a lot of informations on the structure of the module M. For instance, M is a *Cohen-Macaulay* (CM for short) module if and only if $I_M(\mathfrak{q}) = 0$ for some parameter ideal $\mathfrak{q}$ or M is *Buchsbaum* module (see [S-V]) if and only if $I_M(\mathfrak{q})$ is a constant for all parameter ideals $\mathfrak{q}$ of M. In this note we shall investigate this difference, but in a more general situation as follows: Let $x = \{x_1, \cdots, x_d\}$ be a system of parameters (s.o.p. for short) on M and $n = (n_1, \cdots, n_d)$ a d-tuple of positive integers. We consider the difference

$$I_M(n; x) = l(M/(x_1^{n_1}, \cdots, x_d^{n_d})M) - n_1 \cdots n_d e(x; M)$$

as a function in n. In general, this function is not a polynomial in n, even in the case $n_1 = n_2 = \cdots = n_d = t$ (see [G-K]). The necessary and sufficient conditions in term of x, for this function to be a polynomial, have been examined in [C_1]. Here we shall show that the least degree of all polynomials in n bounding above $I_M(n; x)$ is independent of the choice of x (Theorem 2.3). This numerical invariant of M will be called the *polynomial type* of M. The aim of this note is to study the polynomial type of a module over a local ring. In Section 2 we define the polynomial

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