K. Shikishima-Tsuji and M. Katsura
Nagoya Math. J.
Vol. 125 (1992), 93-103

HYPERTRANSCENDENTAL ELEMENTS OF A FORMAL POWER-SERIES RING OF POSITIVE CHARACTERISTIC

KAYOKO SHIKISHIMA-TSUJI AND MASASHI KATSURA

§ 0. Introduction

Throughout this paper, we denote by **N**, **Q** and **R** the set of all natural numbers containing 0, the set of all rational numbers, and the set of all real numbers, respectively.

Let K be a fixed field of positive characteristic p and K_a an algebraic closure of K. We denote by $K[\![X]\!]$ the formal power-series ring and by $d = (d_\mu;\ \mu \in \mathbf{N})$ the formal derivation of $K[\![X]\!]$, i.e., for every $A = \sum_{i=0}^{\infty} a_i X^i \in K[\![X]\!]$, the μ-th derivative $d_\mu A$ of A is defined by

$$d_\mu A = \sum_{i=\mu}^{\infty} \binom{i}{\mu} a_i X^{i-\mu}.$$

For differential rings and differential fields of positive characteristic, see Okugawa [4].

This paper contains three theorems. Let A be an element $\sum_{i=0}^{\infty} a_i X^i$ of $K[\![X]\!]$. We say that A is *hypertranscendental* over K, if, for every $\mu \in \mathbf{N}$, $A, d_1 A, \cdots, d_\mu A$ are algebraically independent over $K(X)$. When the characteristic of the field is zero, the existence of hypertranscendental elements is well-known (see D. Hilbert [1], O. Hölder [2], F. Kuiper [3]). Theorem 1 shows the existence of hypertranscendental elements in case of positive characteristic.

Let L be a differential field and S a subset of a differential extension field of L. We say that S is *differentially independent* over L or all the elements of S are *differentially independent* over L, if for every $\mu \in \mathbf{N}$ and elements $s_1, \cdots, s_\mu$ of S, there are no nonzero differential polynomial $F(X_1, \cdots, X_\mu) \in L\{X_1, \cdots, X_\mu\}$ such that $F(s_1, \cdots, s_\mu) = 0$.

Theorem 2 states that there are infinitely many hypertranscendental

Received February 21, 1991.