C. P. Wang
Nagoya Math. J.
Vol. 125 (1992), 53-72

SURFACES IN MÖBIUS GEOMETRY

CHANGPING WANG*

To Chiu and Ya-Ya

§ 0. Introduction

Our purpose in this paper is to give a basic theory of Möbius differential geometry. In such geometry we study the properties of hypersurfaces in unit sphere S^n which are invariant under the Möbius transformation group on S^n.

Since any Möbius transformation takes oriented spheres in S^n to oriented spheres, we can regard the Möbius transformation group G_n as a subgroup MG_n of the Lie transformation group on the unit tangent bundle US^n of S^n. Furthermore, we can represent the immersed hypersurfaces in S^n by a class of *Lie geometry hypersurfaces* (cf. [9]) called Möbius hypersurfaces. Thus we can use the concepts and the techniques in Lie sphere geometry developed by U. Pinkall ([8], [9]), T. Cecil and S. S. Chern [2] to study the Möbius differential geometry.

We will study in detail the surface theory in Möbius geometry. The same method can be easily generalized to high dimensional cases. We give a complete Möbius invariant system for any immersed surface without umbilic point in S^3 which determines this surface up to Möbius transformations. Moreover, given any such Möbius invariant system we can obtain the corresponding Möbius surface by solving a linear PDE determined by this invariant system.

An immediate application of our theory is the classification of Dupin surfaces in E^3 under the conformal transformation group. We show that up to the conformal transformations a Dupin surface in E^3 is a part of a revolution torus, a right circular cylinder or a right circular cone.

Möbius geometry has a close relation with the famous Willmore conjecture. An elegant application of Möbius geometry was given by R. L.

Received March 27, 1991.
*Supported by Technische Universität Berlin.