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SYMMETRIC FORMS, IDEMPOTENTS AND INVOLUTARY ANTIISOMORPHISMS

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Introduction

Let G be a finite group, F a field and M an irreducible $F[G]$-module. By $\hat{}$ we denote the F-linear involutary antiautomorphism of $F[G]$, induced by inversion on group elements. Suppose that $\operatorname{char}(F) \neq 2$. We then show that M carries a non-singular G-invariant symmetric bilinear form with values in F if and only if there exists a $\hat{}$-invariant idempotent $e \in F[G]$ which generates the projective cover of M. This extends earlier results of W. Willems [Wi]. The assertion is not true if $\operatorname{char}(F) = 2$.

We even consider this question in the class of those finite-dimensional algebras which admit an F-linear involutary antiautomorphism τ and which are symmetric with respect to a τ-invariant symmetric functional. Besides group algebras, also involutary semi-simple F-algebras belong to that class.

In the final part of this paper, we let G be represented irreducibly and orthogonally on a real vector space M. We then show that there is a relationship between G-orbits on the unit sphere of M and idempotents $e \in \mathbb{R}[G]$ such that $M \cong \mathbb{R}[G]e$ and $\hat{e} = e$. This has some connection to a problem in Coding Theory, namely to find G-orbits on the unit sphere whose minimal Euclidian distance is considerably large.

§ 1. Involutary and symmetric algebras

Let A be a finite-dimensional F-algebra over a field F. We set $\overline{A} = A/J(A)$, where $J(A)$ denotes the Jacobson Radical of A. In the following, each A-module should be understood as a finitely generated A-*left*-module.

1.1 LEMMA. *Let* $e, f \in A$ *be primitive idempotents such that* $\overline{ef} \neq 0$. *Then the following assertions hold.*

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