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ON ε-APPROXIMATE SINGULARITIES OF AUTONOMOUS SYSTEMS OF VORTEX TYPE

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§ 0. Introduction

Let us consider three vortex-filaments $z_j(t)$ with strength Γ_j ($j = 1, 2, 3$) in the complex plane $\mathbf{C}$. Then the system of motion equations is given by

$$\text{(E)} \qquad \frac{dz_j}{dt} = \sqrt{-1} \sum_{\substack{k=1 \\ (k \neq j)}} \frac{\Gamma_k}{\bar{z}_j - \bar{z}_k} \quad (j = 1, 2, 3)\,.$$

This system (E) is defined on $V = \mathbf{C}^3 - \Delta$, where $\Delta = \{(z_1, z_2, z_3) \in \mathbf{C}^3; z_j = z_k$ for $j \neq k\}$ is the super-diagonal set of $\mathbf{C}^3$. Let Sol(E) be the space of all smooth solutions of (E) and let $\psi : V \to \mathrm{Sol}(E)$ be a smooth map defined as follows: For any $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in V$, $\psi(\alpha)$ is the solution with initial values α.

It is well-known (cf. [2], p. 260) that if three points α_j of $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ make a regular triangle in $\mathbf{C}$, then $\psi(\alpha)$ becomes a rotational motion about these center of mass, which is called rigid-rotation. This solution $\psi(\alpha)$ has no singular points (cf. Definition 2.1). Now instead of α, let us take $\alpha(\varepsilon) = \alpha + \varepsilon\beta$ as initial values, where ε is a small parameter and $\beta \in \mathbf{C}^3$. Then using computers, we find that $\psi(\alpha(\varepsilon))$ has a singular point at a time $t = T_0(\varepsilon)$, and that $T_0(\varepsilon)$ seems to approach asymptotically to a $\log(1/\varepsilon) + b$ as $\varepsilon \to 0$, for constants a, b (see Figure). We may set the following problems:

(A) Is it true that $T_0(\varepsilon) \sim a \log(1/\varepsilon) + b$ ($\varepsilon \to 0$)?

(B) If (A) is correct, explain how the above constants a and b are determined from the given differential equations (E).

It doesn't seem that such problems have been treated yet.

In this paper we generalize the motion equations (E) on $\mathbf{C}$ to autonomous systems of vortex type on $\mathbf{C}^m$ defined in § 1. We can also consider

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