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ON NILPOTENT EXTENSIONS OF ALGEBRAIC NUMBER FIELDS I

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Introduction

The lower central series of the absolute Galois group of a field is obtained by iterating the process of forming the maximal central extension of the maximal nilpotent extension of a given class, starting with the maximal abelian extension. The purpose of this paper is to give a cohomological description of this central series in case of an algebraic number field. This description is based on a result of Tate which states that the Schur multiplier of the absolute Galois group of a number field is trivial. We are in a profinite situation throughout which requires some functorial background especially for treating the dual of the Schur multiplier of a profinite group. In a future paper we plan to apply our results to construct a nilpotent reciprocity map.

§ 1. Central extensions and Schur multipliers

Let k be an algebraic number field of finite degree over the rationals $\mathbf{Q}$, and let k^{ab} (resp. k^{nil}) be the maximal abelian (resp. nilpotent) extension of k in the algebraic closure $\bar{\mathbf{Q}}$ of $\mathbf{Q}$. For each positive integer c denote by $k^{(c)}/k$ the maximal nilpotent extension of class (at most) c. Hence $k^{(1)} = k^{\mathrm{ab}}$ and $k^{\mathrm{nil}} = \bigcup_{c=1}^{\infty} k^{(c)}$. For convenience we set $k^{(0)} = k$. Put $G^c = \mathrm{Gal}(k^{(c)}/k)$ and $N^c = \mathrm{Gal}(k^{(c)}/k^{(c-1)})$; N^c is a closed normal subgroup of G^c which is contained in the center $Z(G^c)$. Therefore we have a central extension of Galois groups

$$1 \longrightarrow N^{c+1} \longrightarrow G^{c+1} \longrightarrow G^c \longrightarrow 1 .$$

We furnish the rational torus group $T = \mathbf{Q}/\mathbf{Z}$ with the discrete topology and consider it as a Galois module with trivial action.

PROPOSITION 1. *For each $c \geqq 1$, the compact group N^{c+1} is canonically*

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