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MANSFIELD AND SOLOVAY TYPE RESULTS ON COVERING PLANE SETS BY LINES

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F. van Engelen, K. Kunen, and A. W. Miller proved, in [EKM], that for every analytic ($\boldsymbol{\Sigma}_1^1$) set A on the plane, either A can be covered by a countable family of lines or else there is a perfect subset P of A such that no three points of P are collinear. In this paper, we present some generalizations of their result. In particular, a question which was raised by van Engelen et al. in the last paragraph of [EKM] is answered (see Section 3).

We first consider generalizations to κ-Suslin sets and $\boldsymbol{\Sigma}_2^1$ sets on the plane (Section 1). A lightface refinement of the result of van Engelen et al. is also examined (Section 2).

§ 1. Covering a Suslin set

In this section, we prove the following theorem, which is a direct generalization of the theorem of van Engelen et al.

THEOREM 1. *Let $A \subseteq \mathbb{R}^2$ be a κ-Suslin set and let T^0 be the tree associated to a κ-semiscale on A. Then either A can be covered by an $L[T^0]$-definable family of lines with size at most κ, or else there is a perfect subset of A with no three collinear points.*

See Chapter 2 of [Mo] for definitions of "κ-Suslin", "κ-semiscale," etc. We may view Baire's space ${}^\omega\omega$ of irrationals as a subset of the plane $\mathbb{R}^2$ since ${}^\omega\omega$ is homeomorphic to the set of points in $\mathbb{R}^2$ with irrational coordinates. So, $\mathbb{R}^2 - {}^\omega\omega$ is the union of an arithmetically definable family of countably many lines.

For a given κ-Suslin set A, let T^0 be the tree on $\omega \times \kappa$ associated to a κ-semiscale on A. We may suppose, without loss of generality, that $A \subseteq {}^\omega\omega$ and A is the projection of T^0 where the projection $\mathrm{p}[T]$ of a tree

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