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REES ALGEBRAS OF NON-SINGULAR EQUIMULTIPLE PRIME IDEALS*

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Dedicated to Professor H. Matsumura on occasion of his 60th birthday

§ 1. Introduction

In a recent paper [HI2] the first named author and S. Ikeda have investigated Gorenstein properties under blowing up of height two equimultiple ideals from the arithmetical point of view. The problem is to what extent does the Gorenstein property of the Rees algebra $R(\mathfrak{a}) = \bigoplus_{n \geqq 0} \mathfrak{a}^n$ of an equimultiple ideal $\mathfrak{a}$ of a local ring $(A, \mathfrak{m})$ describe the structure of A and $\mathfrak{a}$. One result is that if A is a generalized Cohen-Macaulay ring (i.e. a ring of finite local cohomology) with $\dim(A) \geqq 4$ and if there is an height two equimultiple prime ideal $\mathfrak{p}$ such that $R(\mathfrak{p})$ is Gorenstein, then A is Gorenstein and $\mathfrak{p}$ is generated by a regular sequence (see [HI2], Theorem 2.6). That has led to the question whether this result is still valid for $\dim(A) = 3$. A partial answer was given in [HI2], Proposition 2.10, where "equimultiplicity" was replaced by the stronger condition "$\mathfrak{p}/\mathfrak{p}^2$ is flat over $A/\mathfrak{p}$".

In this paper we extend first Theorem 2.6 in [HI2] to the Cohen-Macaulay case in the following sense, assuming that $\sharp A/m = \infty$.

THEOREM (1.1). *Let $(A, \mathfrak{m})$ be a generalized Cohen-Macaulay ring with $\dim(A) \geqq 4$. Assume that there is a height two equimultiple prime ideal $\mathfrak{p}$ of A such that*

(i) *$A_\mathfrak{p}$ is regular*

(ii) *$R(\mathfrak{p})$ is Cohen-Macaulay.*

Then A is a Cohen-Macaulay ring and $\mathfrak{p}$ is generated by a regular sequence.

Since A_p is regular if $R(\mathfrak{p})$ is Gorenstein [I], we obtain Theorem 2.6

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