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Nagoya Math. J.
Vol. 123 (1991), 171-201

# IDEALS OF BOUNDED HOLOMORPHIC FUNCTIONS ON SIMPLE $n$-SHEETED DISCS

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## § 1. Introduction

**1.1.** As usual we denote by $H^\infty(R)$ the Banach algebra of bounded holomorphic functions on a Riemann surface $R$ equipped with the supremum norm $\|\cdot\|_\infty$. Consider the ideal $I(f_1, \cdots, f_m)$ of $H^\infty(R)$ generated by functions $f_1, \cdots, f_m$ in $H^\infty(R)$. If a function $g$ in $H^\infty(R)$ belongs to $I(f_1, \cdots, f_m)$, or equivalently, if there exist $m$ functions $h_1, \cdots, h_m$ in $H^\infty(R)$ with

$$\sum_{j=1}^{m} f_j h_j = g$$

on $R$, then common zero points of $f_1, \cdots, f_m$ are also zero points of $g$ in the following strong sense:

$$\sum_{j=1}^{m} |f_j|^2 \geq \delta |g|^2 \tag{1.1}$$

on $R$ for a positive constant $\delta > 0$. The *generalized corona problem* asks whether the converse is valid or not. In the case $g \equiv 1$ on $R$ the problem is referred to simply as the *corona problem.*

The simple corona problem was solved by L. Carleson in [C1] in the case where $R$ is the open unit disc $D$ (see, further [K], [G3], [Ga], [BR], [S1], etc). Whether the result can be generalized to arbitrary plane regions is still open, but there are many plane regions for which the problem is in the affirmative (cf. [St1], [B1], [B2], [C2], [GJ], [G1], [Z], [D], [DW], [Na], [M], etc). While there is an example by B. Cole showing that the problem is in the negative for general Riemann surfaces (cf. [G2], [N1]). Also there are many cases in this category where the problem is in the positive (cf. [A1], [A2], [St2], [St3], [Fo], [EM1], [EM2], [H1], [H2], [N2], [JM], etc).

Received January 29, 1991.