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Nagoya Math. J.
Vol. 123 (1991), 153-169

ITO'S FORMULA AND LEVY'S LAPLACIAN II

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Dedicated to Professor Takeyuki Hida

§1. Introduction

The white noise calculus was initiated by T. Hida in 1970 in his Princeton University Mathematical Notes [3]. Recent development of the theory shows that the Laplacian plays an essential role in the analysis in question. Indeed, several kinds of Laplacians should be introduced depending on the choice of the class of white noise functionals to be analysed, as can be seen in [4], [13], [18] and so forth. Among others, we should like to emphasize the importance of the infinite dimensional Laplace-Beltrami operator, Volterra's Laplacian and Lévy's Laplacian (See [13], [18] and [20]).

In this paper, we shall discuss characteristic properties of Lévy's Laplacian Δ_L and some of related topics as well as its applications, in particular, to form explicit solutions of a Schrödinger equation, where the Laplacian naturally appears.

Following [6] and [15], we shall first introduce, in Section 2, the space $(E)^*$ of generalized white noise functionals and the usual tools of the analysis like the S-transform, the U-functional, and the $\mathscr{T}$-transform on $(E)^*$. It is noted that Δ_L acts effectively on a certain subspace of $(E)^*$ and it does annihilate ordinary white noise functionals. We then come to the calculus of $(E)^*$-functionals in terms of the white noise $\dot{B}(t)$, $t \in T$, (T is an interval) which is now thought of as a member of the variables of white noise functionals.

We establish eigenfunctionals of Δ_L and deal with the heat equation satisfied by the expectation functional of the delta functional in Section 3. Lévy's group and an algebra generated by infinitesimal generators of the infinite dimensional rotation group are dealt with to some extent

Received February 25, 1991.