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ON A CLASS OF INSOLUBLE BINARY QUADRATIC DIOPHANTINE EQUATIONS

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§ 0. Introduction

The binary quadratic diophantine equation

$$|x^2 - ny^2| = t$$

is of interest in the class number problem for real quadratic number fields and was studied in recent years by several authors (see [4], [5], [2] and the literature cited there).

To be precise, for a positive square-free integer n, we set

$$\sigma_n = \begin{cases} 1, & \text{if } n \not\equiv 1 \bmod 4, \\ 2, & \text{if } n \equiv 1 \bmod 4; \end{cases}$$

a solution $(x, y) \in \mathbf{Z}$ of the diophantine equation

$$|x^2 - ny^2| = \sigma_n^2 t$$

is called *primitive*, if $(x, y) | \sigma_n$, where (x, y) denotes the g.c.d. of x and y. The reason for this terminology will become clear from the theory of quadratic orders, to be explained in § 1.

R. A. Mollin [4] proved, generalizing previous results by Yokoi [5] and others, the following criterion.

PROPOSITION 0. *Let s, t, r be integers such that $n = (st)^2 + r > 5$ is squarefree and the following conditions are satisfied*:

(1) $s \geq 1$, $t \geq 2$ *and* $(t, r) = 1$;

(2) $r | 4s$, *and* $-st < r \leq st$;

(3) *If* $n \equiv 1 \bmod 4$, *then* $|r| \in \{1, 4\}$.

(4) *If* $|r| = 4$, *then* $s \geq 2$.

(5) *If* $r = 1$, *then* $s \geq 3$ *and* $2 | st$.

Then the diophantine equation $|x^2 - ny^2| = \sigma_n^2 t$ *has a primitive solution if*

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