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# DISTRIBUTIONS OF STABLE RANDOM FIELDS OF CHENTSOV TYPE

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## § 1. Introduction

In this paper we discuss the determinism of distributions of some stable random fields which are constructed through integral-geometric method. The determinism depends on the dimension of the parameter space $R^d$.

We say that a family of random variables $\{X(t); t \in \mathbf{R}^d\}$ is a *symmetric α-stable* (abbreviated to $S\alpha S$) random field on $\mathbf{R}^d$ if every finite linear combination $\sum_{i=1}^n a_i X(t_i)$ has a symmetric stable distribution of index $\alpha$. Let $(E, \mathscr{B}, \mu)$ be a measure space. We say that a family of random variables $\{Y(B); B \in \mathscr{B}, \mu(B) < \infty\}$ is the *$S\alpha S$ random measure* corresponding to $(E, \mathscr{B}, \mu)$ if (i) $E(\exp[izY(B)]) = \exp(-\mu(B)|z|^\alpha)$, for $z \in \mathbf{R}$ and $\mu(B) < \infty$, (ii) $Y(B_1)$, $Y(B_2)$, $\cdots$ are independent whenever $B_1, B_2, \cdots$ are disjoint and $\mu(B_j) < \infty$, $j = 1, 2, \cdots$, (iii) $Y(\bigcup_{j=1}^\infty B_j) = \sum_{j=1}^\infty Y(B_j)$ a.s. whenever $B_1, B_2, \cdots$ are disjoint and $\mu(\bigcup_{j=1}^\infty B_j) < \infty$.

We define a class of $S\alpha S$ random fields with a particular choice of $E$. Let $E_0$ be the set of all $(d-1)$-dimensional spheres in $\mathbf{R}^d$. Any element of $E_0$ is expressed by a coordinate system $(r, x)$, where $(r, x)$ corresponds to the sphere with radius $r \in \mathbf{R}_+ = (0, \infty)$ and center $x \in \mathbf{R}^d$. Thus we make the identification

$$(1.1) \qquad E_0 = \{(r, x); r \in \mathbf{R}_+, x \in \mathbf{R}^i\}.$$

For $t \in \mathbf{R}^d$, let $s_t$ be the set of all spheres which separate the point $t$ and the origin $O$, namely

$$(1.2) \qquad S_t = \{(r, x); d(x, O) \leqq r\} \triangle \{(r, x); d(r, x) \leqq r\},$$

where $A \triangle B$ denotes the symmetric difference of $A$ and $B$ and $d(a, b)$ denotes the Euclidean distance between $a$ and $b$. Let $\mathscr{B}_0$ be the $\sigma$-algebra

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