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HARMONIC ANALYSIS ON THE QUOTIENT SPACES OF HEISENBERG GROUPS

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A certain nilpotent Lie group plays an important role in the study of the foundations of quantum mechanics ([Wey]) and of the theory of theta series (see [C], [I] and [Wei]). This work shows how theta series are applied to decompose the natural unitary representation of a Heisenberg group.

For any positive integers g and h, we consider the Heisenberg group

$$H_R^{(g,h)} := \{[(\lambda, \mu), \kappa] \mid \lambda, \mu \in R^{(h,g)},\ \kappa \in R^{(h,h)},\ \kappa + \mu\,{}^t\lambda \text{ symmetric}\}$$

endowed with the following multiplication law

$$[(\lambda, \mu), \kappa] \circ [(\lambda', \mu'), \kappa'] = [(\lambda + \lambda', \mu + \mu'), \kappa + \kappa' + \lambda\,{}^t\mu' - \mu\,{}^t\lambda'] .$$

The mapping

$$H_R^{(g,h)} \ni [(\lambda, \mu), \kappa] \longrightarrow \begin{pmatrix} E_g & 0 & 0 & {}^t\mu \\ \lambda & E_h & \mu & \kappa \\ 0 & 0 & E_g & -{}^t\lambda \\ 0 & 0 & 0 & E_h \end{pmatrix}$$

defines an embedding of $H_R^{(g,h)}$ into the symplectic group $Sp(g+h, R)$. We refer to [Z] for the motivation of the study of this Heisenberg group $H_R^{(g,h)}$. $H_Z^{(g,h)}$ denotes the discrete subgroup of $H_R^{(g,h)}$ consisting of integral elements, and $L^2(H_Z^{(g,h)}\backslash H_R^{(g,h)})$ is the L^2-space of the quotient space $H_Z^{(g,h)}\backslash H_R^{(g,h)}$ with respect to the invariant measure

$$d\lambda_{11} \cdots d\lambda_{h,g-1} d\lambda_{hg} d\mu_{11} \cdots d\mu_{h,g-1} d\mu_{hg} d\kappa_{11} d\kappa_{12} \cdots d\kappa_{h-1,h} d\kappa_{hh}.$$

We have the natural unitary representation ρ on $L^2(H_Z^{(g,h)}\backslash H_R^{(g,h)})$ given by

$$\rho([(\lambda', \mu'), \kappa'])\phi([(\lambda, \mu), \kappa]) = \phi([(\lambda, \mu), \kappa] \circ [(\lambda', \mu'), \kappa']).$$

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