K. Moriyasu
Nagoya Math. J.
Vol. 123 (1991), 91-102

THE TOPOLOGICAL STABILITY OF DIFFEOMORPHISMS

KAZUMINE MORIYASU

§ 1. Introduction

The present paper is concerned with the stability of diffeomorphisms of C^∞ closed manifolds. Let M be a C^∞ closed manifold and $\mathrm{Diff}^r(M)$ be the space of C^r diffeomorphisms of M endowed with the C^r topology (in this paper we deal with only the case $r = 0$ or 1). Let us define

$$\mathscr{F}(M) = \left\{ f \in \mathrm{Diff}^1(M) \,\middle|\, \begin{array}{l} \text{there exists a } C^1 \text{ neighborhood } \mathscr{U}(f) \text{ of} \\ f \text{ such that all periodic points of every} \\ g \in \mathscr{U}(f) \text{ are hyperbolic} \end{array} \right\}.$$

Then every C^1 structurally stable and Ω-stable diffeomorphism belongs to $\mathscr{F}(M)$ (see [3]). In light of this result Mañé solved in [5] the C^1 Structural Stability Conjecture by Palis and Smale. After that Palis [9] obtained, in proving that every diffeomorphism belonging to $\mathscr{F}(M)$ is approximated by Axiom A diffeomorphisms with no cycle, the C^1 Ω-Stability Conjecture. Recently Aoki [2] proved that every diffeomorphism belonging to $\mathscr{F}(M)$ is Axiom A diffeomorphisms with no cycle (a conjecture by Palis and Mañé). For the topological stability Walters [14] proved that every Anosov diffeomorphism is topologically stable. In [7] Nitecki showed that every Axiom A diffeomorphism having strong transversality is topologically stable, and that every Axiom A diffeomorphism having no cycle is Ω-topologically stable.

Thus it will be natural to ask whether topologically stable diffeomorphisms belonging to $\mathrm{Diff}^1(M)$ satisfy Axiom A and strong transversality.

Let $f \in \mathrm{Diff}^1(M)$. Then $f: M \to M$ is topologically stable if and only if given $\varepsilon > 0$ there exists $\delta > 0$ such that for any $g \in \mathrm{Diff}^0(M)$ with $d(f, g) < \delta$ there exists a continuous map $h: M \to M$ satisfying $h \circ g = f \circ h$ and $d(h, \mathrm{id}) < \varepsilon$ (where id is the identity). Note that if ε is sufficiently small then the above continuous map h is surjective since h is homotopic to id. We denote by $\Omega(f)$ the set of nonwandering points of f. A diffeo-

Received July 20, 1990.