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OPTIMAL CONTROL FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS AND VISCOSITY SOLUTIONS OF BELLMAN EQUATIONS

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§ 1. Introduction

Recently M. G. Crandall and P. L. Lions developed the viscosity theory on nonlinear equations in infinite dimensions and optimal control in Hilbert spaces, in two series of papers, [1], [4].

In this article we will study of optimal control of stochastic partial differential equations and viscosity solutions of Bellman equation (1.1) below,

$$(1.1)\qquad \sup_u\Big(-\frac{1}{2}(D^2\nu(\phi)M\phi, M\phi) - (D\nu(\phi), L(u)\phi) + \lambda\nu(\phi) - F(\phi)\Big) = 0\,,$$

where D and D^2 denote the first and second Fréchet differentials and M and $L(u)$ are the first and second order differential operators respectively (see (2.1)).

Let us consider the following stochastic partial differential equation, (SPDE in short)

$$(1.2)\qquad \begin{aligned} dq(t) &= \sum_{ij=0}^{d}\frac{\partial}{\partial x_i}\Big(a^{ij}(x, U(t))\frac{\partial}{\partial x_j}q(t, x) + f^i(x, U(t))\Big)dt \\ &\quad + \sum_{k=1}^{m}\Big(\sum_{i=0}^{d} b_k^i(x)\frac{\partial}{\partial x_i}q(t, x) + g_k(x)\Big)dW^k(t)\,, \end{aligned}$$

where $W = (W^1, \cdots, W^m)$ is an m-dimensional standard Wiener process and $U(t)$ an admissible control. We will define the criterion J by

$$(1.3)\qquad J(\phi, U) = E\int_0^\infty e^{-\lambda t}F(q(t, \phi, U))dt$$

where $q(t, \phi, U)$ denotes a solution of (1.2) starting at ϕ. The function V defined by

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