Y. Takeda
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FIBRATIONS WITH MOVING CUSPIDAL SINGULARITIES

YOSHIFUMI TAKEDA

Let f: $V \to C$ be a fibration from a smooth projective surface onto a smooth projective curve over an algebraically closed field k. In the case of characteristic zero, almost all fibres of f are nonsingular. In the case of positive characteristic, it is, however, known that there exist fibrations whose general fibres have singularities. Moreover, it seems that such fibrations often have pathological phenomena of algebraic geometry in positive characteristic (see M. Raynaud [7], W. Lang [4]).

In the present article, we consider the surfaces with cuspidal fibration which are obtained as the quotients of surfaces with smooth fibration by p-closed rational vector fields. In particular, we shall give a construction of generalized Raynaud surfaces and give a dimensional estimate of the nonzero first cohomology group which appears in counter-examples to the Kodaira Vanishing Theorem in positive characteristic.

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§ 1. Preliminaries

Throughout this article, we assume that k is an algebraically closed field of characteristic $p \geq 3$. Let V be a smooth projective surface over k and let D be a k-derivation of the function field $k(V)$. Then we say that D is a *rational vector field* on V. We call D a *p-closed* rational vector field if there exists a rational function h on V such that $D^p = hD$. For a rational vector field D, let V^D be the scheme whose underlying space is the same as V and whose structure sheaf consists of the germs of sections of $\mathcal{O}_V$ killed by D. We call V^D the *quotient* of V by D. Then the quotient V^D is normal and the canonical projection π: $V \to V^D$ is a purely inseparable morphism. Moreover, if D is p-closed, then the degree of π is p. Let (x, y) be a local coordinate system at a point P of V.

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