

**EXISTENCE AND NON-EXISTENCE OF NULL-SOLUTIONS  
 FOR SOME NON-FUCHSIAN PARTIAL DIFFERENTIAL  
 OPERATORS WITH  $T$ -DEPENDENT COEFFICIENTS**

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*Dedicated to Professor S. Matsuura on his 60th birthday*

**§0. Introduction**

Since M.S. Baouendi and C. Goulaouic ([2], [3]) defined partial differential operators of *Fuchs type* and proved theorems of Cauchy-Kowalevskaya type and Holmgren type, many authors have investigated operators of Fuchs type in various categories, that is, real-analytic,  $C^\infty$  and so on. (Cf. [1], [4], [6], [8], [9], [11], [12], [17], [18], [19], [20], [21] etc.)

**DEFINITION 0.1.** A partial differential operator  $P$  is called of *Fuchs type* (or *Fuchsian*) with weight  $m - k$  ( $0 \leq k \leq m$ ), when  $P$  has the following form:

$$(0.1) \quad P = t^k \partial_t^m + a_1(x) t^{k-1} \partial_t^{m-1} + \cdots + a_k(x) \partial_t^{m-k} \\ + \sum_{\substack{j+|\alpha| \leq m \\ j \leq m-1}} t^{\max(0, j+k-m+1)} a_{j,\alpha}(t, x) \partial_t^j \partial_x^\alpha,$$

where  $a_j(x)$ ,  $a_{j,\alpha}(t, x)$  are smooth, that is, real-analytic,  $C^\infty$  and so on. (Notations are given later.)

**Remark 0.2.** Note that the operator  $P$  is Fuchsian with weight  $m - k$  if and only if  $t^{m-k} P$  is Fuchsian with weight 0.

It has become known that Fuchsian operators have various "good" properties. Among them, we are concerned with the following uniqueness property. (See also [17].)

**THEOREM 0.3** ([2]). *If  $P$  is Fuchsian with real-analytic coefficients, then there exists a positive integer  $N$  depending on  $P$  such that the following holds:*

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Received March 7, 1990.