S. Matsumoto and A. Sato
Nagoya Math. J.
Vol. 122 (1991), 75-82

PERTURBATION OF THE HOPF FLOW AND TRANSVERSE FOLIATIONS

SHIGENORI MATSUMOTO AND ATSUSHI SATO

Dedicated to Professor Ken'ichi Shiraiwa on his 60th birthday

§1. Introduction

Consider a nonsingular vector field X on a closed manifold M^n. As a matter of fact, X always admits a transverse codimension one plane field, which however may fail to be integrable. In fact it is well known that there are many examples of vector fields which do not admit transverse foliations.

The purpose of this paper is to contribute to the study of the topology of the set of vector fields which admit transverse foliations. In what follows, we only consider the three dimensional case, since there the qualitative study of foliations is a rather mature stage and provides us with powerful tools. For example, a complete criterion for admitting transverse foliations is obtained for 3-dimensional Smale flows by Goodman ([G1], [G2]).

Denote by $NS(M^3)$ the set of nonsingular C^r verctor fields $(1 \leq r < \infty)$ on a 3-manifold M^3 and by $\pitchfork(M^3)$ the subset of those vector fields which admit transverse codimension one C^1 foliations. Andrade ([A]), working with the C^0 topology on $NS(M^3)$, has shown that for any 3-manifold M^3,

$$\pitchfork(M^3) \subsetneqq \mathrm{Int}\,\overline{\pitchfork(M^3)}\,,$$

where he made use of Goodman's criterion and the fact that Smale flows are C^0 dense in $NS(M^3)$.

Our theme here is to consider the same problem in the context of the C^r topology. Henceforth in this paper we assume that *the set $NS(M^3)$ of nonsingular C^r vector fields is equipped with the C^r topology* $(1 \leq r < \infty)$. Obviously $\pitchfork(M^3)$ is an open subset of $NS(M^3)$.

Received February 2, 1990.