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NOTES ON ENERGY FOR SPACE-TIME PROCESSES OVER LÉVY PROCESSES

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Dedicated to Professor Masanori Kishi on his 60th birthday

§ 1. Introduction

Let $X = (X_t, 0 \leq t < \infty)$ be a Lévy process on the Euclidean space R^d, that is, a process on R^d with stationary independent increments which has right continuous paths with left limits. We denote by P^x the probability measure such that $P^x(X_0 = x) = 1$ and by E^x the expectation relative to P^x. The process is characterized by the exponent Ψ through

$$E^0(\exp i\langle z, X_t\rangle) = \exp(-t\Psi(z)) .$$

The λ-energy $E_X^\lambda(\nu)$ of a measure ν on R^d for X is defined by

$$E_X^\lambda(\nu) = \int \mathrm{Re}([\lambda + \Psi(z)]^{-1})|\mathscr{F}\nu(z)|^2 dz ,$$

where $\mathscr{F}$ denotes the Fourier tranform on R^d. A nice explanation of the reason why it is called the λ-energy is given in Rao [11]. Throughout the paper $\mathscr{F}\nu(z)$ is defined by $\int \exp i\langle z, x\rangle\ \nu(dx)$ and we write $\mathscr{F}u(z)$ in place of $\mathscr{F}udx(z)$ if $\nu(dx) = u(x)dx$. So our λ-energy differs from Rao's by a constant multiple.

The space-time process $Y = (Y_t, 0 \leq t < \infty)$ over X is a Lévy process on $R^1 \times R^d$ defined on the probability space $(R^1 \times \Omega, P^{r,x})$, where Ω is the path space of X and $P^{r,x} = \delta_r \otimes P^x$, δ_r being the Dirac measure at $r \in R^1$. The trajectory $Y_t(r, \omega)$ is $(r + t, X_t(\omega))$ and the exponent of Y is $\Psi(z) - it$. So the λ-energy $E_Y^\lambda(\mu)$ of a measure μ on $R^1 \times R^d$ for Y is

$$E_Y^\lambda(\mu) = \iint \mathrm{Re}([\lambda + \Psi(z) - it]^{-1})|\mathscr{F}\mu(t, z)|^2 dt\, dz ,$$

where $\mathscr{F}$ denotes the Fourier transform on $R^1 \times R^d$.

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