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ON THE GRADE AND COGRADE OF A NOETHERIAN FILTRATION

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§1. Introduction

All rings in this paper are assumed to be commutative with identity and the terminology is standard.

Filtrations are a useful generalization of the sets of powers of an ideal I in a ring R, and there are many important filtrations that are generally not such powers of an ideal. (For example: $\{Q^{(n)}\}_{n\geq 0}$, where Q is a primary ideal; $\{(I^n)_a\}_{n\geq 0}$, where $(I^n)_a$ is the integral closure in R of I^n; and, $\{u^n\mathscr{B} \cap R\}_{n\geq 0}$, where $\mathscr{B}$ is a graded subring of $R[u, t]$ that contains $R[u, tI]$ and I is a given ideal of R.) They have played an important role in many research papers, and there are many results concerning them in the literature.

In several recent papers a number of important theorems concerning ideals in a Noetherian ring have been extended to Noetherian filtrations (e.g., see [1, 13, 14, 15, 25, 28, 29]). And in [6, 26] a number of results concerning the asymptotic prime divisors of an ideal are extended to finite collections of ideals. The results in this paper combine both types of extensions; specifically, we extend the "asymptotic" definitions (for an ideal) to a collection of $g \geq 1$ Noetheian filtrations. (As in the ideal case, it turns our that when working with filtrations $\phi_1, \cdots, \phi_g$ (with $g > 1$), for the asymptotic prime divisor case it must be assumed that each $\phi_i(1)$ has height at least one, and for the essential prime divisor case the corresponding assumption is that each $\phi_i(1)$ is regular.) Our results then imply, as a special case, that the corresponding results hold for finite collections of ideals.

To be more specific, in this paper we extend the definitions of four types of prime divisors (viz, asymptotic, essential, quintasymptotic, and quintessential) from a single ideal to a collection $\Phi = (\phi_1, \cdots, \phi_g)$ of $g \geq 1$

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