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WHITE NOISE ANALYSIS AND TANAKA FORMULA FOR INTERSECTIONS OF PLANAR BROWNIAN MOTION

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§ 1. Introduction

In this paper, we shall use Hida's [5, 7, 9] theory of generalized Brownian functionals (or named white noise analysis) to establish a stochastic integral formula concerning the multiple intersection local times of planar Brownian motion $B(t)$. First, we review some basic facts. It is well-known that, for each $r \geqslant 2$, a.s. $B(t)$ has points of multiplicity r, see Dvoretzky-Erdös-Kakutani [1] and Geman-Horowitz-Rosen [4]. A formal measure on such r-fold self-intersections is

$$\int \cdots_{D} \int \delta(B(t_2) - B(t_1)) \cdots \delta(B(t_r) - B(t_{r-1}))dt_1 \cdots dt_r, \tag{1.1}$$

where $D \subset \{(t_1, \cdots, t_r) | 0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_r < \infty\}$. Rosen [15] proved that (a.s.) there exists measurable $\alpha_r(x, D)$, $x \in R^{2(r-1)}$, called the r-fold intersection local time, such that for all bounded measurable $g(x)$ on $R^{2(r-1)}$

$$\int \cdots_{D} \int g(B(t_2) - B(t_1), \cdots, B(t_r) - B(t_{r-1}))dt_1 \cdots dt_r$$
$$= \int_{R^{2(r-1)}} \cdots \int g(x)\alpha_r(x, D)dx .$$

In case $D = \prod_{i=1}^{r} [a_i, b_i]$, $0 < a_1 < b_1 < a_2 < b_2 < \cdots < b_r$ (the off-diagonal case), Rosen [15, 17] proved that (a version of) $\alpha_r(x, D)$ can be chosen so that it is (a.s.) jointly continuous in x, a_i, b_i. Thus, (1.1) is represented as $\alpha_r(0, D)$ in this case. However, in case $D = D_T = \{(t_1, \cdots, t_r) | 0 \leqslant t_1 \leqslant \cdots t_r \leqslant T\}$ (the diagonal case), then $\alpha_r(x, T) \equiv \alpha_r(x, D_T)$ can only be continuous on $(R^2 \backslash \{0\})^{r-1}$. To study the asymptotics of $\alpha_r(x, T)$ as $x \to 0$, it is intended to find that after substracting off certain explicit "infinite part" from $\alpha_r(x, T)$ the remainder $\tilde{\alpha}_r(x, T)$, called the renormalized intersection local time, admits a jointly continuous extension to all $(x, T) \in R^{2(r-1)} \times R_+$.

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