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A CHARACTERIZATION OF WHITE NOISE TEST FUNCTIONALS

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§ 1. Introduction and main result

In a recent paper [PS 89], two of the present authors have found a characterization of a certain space $(\mathscr{S})^*$ of generalized functionals of white noise, i.e. generalized functionals on $\mathscr{S}'(\mathbb{R})$ equipped with the σ-algebra $\mathscr{B}$ generated by its cyclinder sets and with the white noise measure μ given by

$$\int_{\mathscr{S}'(\mathbb{R})} \exp(i\langle x, \xi\rangle) d\mu(x) = \exp\left(-\frac{1}{2}|\xi|_2^2\right),$$

for $\xi \in \mathscr{S}(\mathbb{R})$. Here, $|\cdot|_2$ denotes the norm of $L^2(\mathbb{R})$, and $\langle\cdot, \cdot\rangle$ dual pairing. Below, we shall shortly recall the construction of the space $(\mathscr{S})^*$ as the dual of a space $(\mathscr{S})$ of "smooth" functionals on $\mathscr{S}'(\mathbb{R})$. The characterization mentioned above is of considerable power: it provides an extremely convenient way to decide whether a certain given functional is an element in $(\mathscr{S})^*$. This has been shown in [PS 89] for a number of examples (especially for certain measures on $\mathscr{S}'(\mathbb{R})$). The purpose of the present note is to give a similar characterization for the elements $(\mathscr{S})$ of test functionals. For notation, definitions, more background and references, we refer the reader to [PS 89].

Let $\Gamma(A)$ denote the second quantization of the self-adjoint $L^2(\mathbb{R})$-operator A which on $\mathscr{S}(\mathbb{R})$ is defined as

$$A\xi(u) = -\xi''(u) + (1 + u^2)\xi(u), \qquad \xi \in \mathscr{S}(\mathbb{R}),\ u \in \mathbb{R}.$$

Let $\mathscr{P}$ denote the algebra of smooth polynomials on $\mathscr{S}'(\mathbb{R})$, i.e. $\mathscr{P}$ is generated by the random variables $X_\xi = \langle\cdot, \xi\rangle$, $\xi \in \mathscr{S}(\mathbb{R})$. For $p \geq 0$, let $\mathscr{S}_p(\mathbb{R})$ denote the completion of $\mathscr{S}(\mathbb{R})$ with respect to the norm $|\xi|_{2,p} = |A^p\xi|_2$, and let $(\mathscr{S})_p$ denote the completion of $\mathscr{P}$ with respect to the norm

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