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ON GENERALIZED WHITTAKER FUNCTIONS ON SIEGEL'S UPPER HALF SPACE OF DEGREE 2

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Dedicated to Professor Tomio Kubota on his sixtieth birthday

In [5], H. Maass showed that the dimension of a space of generalized Whittaker functions satisfying certain system of differential equations on Siegel's upper half space H_2 of degree 2 is three. First of all, we shall investigate the structure of a space of generalized Whittaker functions which are eigen functions for the algebra of invariant differential operators on H_2. The theory of generalized Whittaker functions is discussed in Yamashita [12], [13], [14], [15] with full generality. But, we will get an outlook of the space of generalized Whittaker functions by using elementary calculus instead of representation theory of Lie groups. Generalized Whittaker functions, naturally appear in the theory of indefinite theta function, and so we shall next show commutation relations between the invariant differential operators on H_2 and those on the product $H_1 \times H_1$ of two copies of the upper half plane H_1 operated on a theta function. The relations are analogies of commutation relations for Hecke operators in [1], [16], [17] and are proved in some cases with the Laplacian in [8], [2]. We essentially use the result in Nakajima [10] where the generators of the center of the universal enveloping algebra of $\mathfrak{sp}(2, \mathbb{R})$ are explicitly given. By commutation relations we can construct an automorphic form F on H_2 corresponding to an L-function with Grössencharacter of a certain biquadratic field. Generalized Whittaker functions investigated in the present paper appear in the Fourier expansion of F with respect to translations in H_2 and so we can define the "constant part" of the Fourier coefficient as the ratio of the Fourier coefficient to a generalized Whittaker function. The constant part of a certain Fourier coefficient of an automorphic form analogous to F is given in 2. (See in particular (2.10).)

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