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A GENERALIZATION OF HILBERT'S THEOREM 94

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In this paper we shall prove the following theorem conjectured by Miyake in [3] (see also Jaulent [2]).

THEOREM. *Let k be a finite algebraic number field and K be an unramified abelian extension of k, then all ideals belonging to at least $[K: k]$ ideal classes of k become principal in K.*

Since the capitulation homomorphism is equivalently translated to a group-transfer of the galois group (see Miyake [3]), it is enough to prove the following group-theoretical verison:

THEOREM (The group-theoretical version). *Let H be a finite group and N be a normal subgroup of H containing the commutator subgroup H^c of H. Then $[H: N]$ divides the order of the kernel of the group-transfer $V_{H\to N}: H^{ab} \to N^{ab}$.*

Hilbert's theorem 94 and the principal ideal theorem immediately follow from our theorem.

§ 1. Notations and two lemmas

For a group H, we denote the commutator group of H by H^c, and the augmentation ideal of the integral group algebra $\mathbf{Z}[H]$ by I_H. Put also

$$H^{ab} = H/H^c\,,$$
$$\mathrm{Tr}_H = \sum_{g\in H} g \in \mathbf{Z}[H]\,,$$

and

$$A_H = \mathbf{Z}[H]/(\mathrm{Tr}_H)\,.$$

For a $\mathbf{Z}[H]$-module M, we denote the $\mathbf{Z}[H]$-submodule consisting of all the H-invariant elements of M by M^H and the Pontrjagin dual of M by

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