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ON THE TOPOLOGY OF FULL NON-DEGENERATE COMPLETE INTERSECTION VARIETY

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§ 1. Introduction

Let $h_1(\mathbf{u}), \cdots, h_k(\mathbf{u})$ be Laurent polynomials of m-variables and let

$$Z^* = \{\mathbf{u} \in \mathbf{C}^{*m};\ h_1(\mathbf{u}) = \cdots = h_k(\mathbf{u}) = 0\}$$

be a non-degenerate complete intersection variety. Such an intersection variety appears as an exceptional divisor of a resolution of non-degenerate complete intersection varieties with an isolated singularity at the origin (Ok4]). We say that Z^* is *full* if $\dim(\Delta(h_\alpha)) = m$ for any $\alpha = 1, \cdots, k$. Let I be a subset of $\{1, \cdots, m\}$. We say that Z^* is *I-full* if (i) for each $\alpha = 1, \cdots, k$, $h_\alpha(\mathbf{u})$ is a polynomial in the variables $\{u_i;\ i \in I\}$ (fixing other variables) and (ii) for any $J \supset I^c$, the polynomials $\{h_\alpha^J(\mathbf{u}_J);\ \alpha = 1, \cdots, k\}$ are not constantly zero and the *variety* $\{\mathbf{u}^J \in \mathbf{C}^{*J};\ h_1^J(\mathbf{u}_J) = \cdots = h_k^J(\mathbf{u}_J) = 0\}$ is full in the above sense where h_α^J is the restriction of h_α to the coordinate subspace $\mathbf{C}^J = \{\mathbf{u} \in \mathbf{C}^m;\ u_i = 0 \text{ if } i \notin J\}$ and I^c is the complement of I in $\{1, \cdots, m\}$. Thus any full non-degenerate complete intersection variety is $\emptyset$-full. Assume that Z^* is I-full and let

$$Z = \{\mathbf{u} \in \mathbf{C}^I \times \mathbf{C}^{*I^c};\ h_1(\mathbf{u}) = \cdots = h_k(\mathbf{u}) = 0\}.$$

Here we identify $\mathbf{C}^I \times \mathbf{C}^{*I^c}$ with the subspace of $\mathbf{C}^m$ by $\mathbf{C}^I \times \mathbf{C}^{*I^c} = \{\mathbf{z} \in \mathbf{C}^m;\ z_i \neq 0,\ i \in I^c\}$. In the case that $I = \{1, \cdots, m\}$, the I-fullness condition implies that each h_α has a non-zero constant term and each $h_\alpha(\mathbf{u})$ is a convenient polynomial. Here the polynomial h_α is called convenient if and only if $h_\alpha^{\{i\}}$ is not constantly zero for any $1 \leq i \leq m$. In particular, $\vec{0} \notin Z$ in this case. The purpose of this paper is to study the topology of a full non-degenerate complete intersection variety. We will prove

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