K. Nomizu and F. Podestà
Nagoya Math. J.
Vol. 121 (1991), 127–135

ON THE CARTAN-NORDEN THEOREM FOR AFFINE KÄHLER IMMERSIONS

KATSUMI NOMIZU AND FABIO PODESTÀ

In [N-Pi-Po] the notion of affine Kähler immersion for complex manifolds has been introduced: if M^n is an n-dimensional complex manifold and f: $M^n \to \mathbf{C}^{n+1}$ is a holomorphic immersion together with an antiholomorphic transversal vector field ζ, we can induce a connection ∇ on M^n which is Kähler-like, that is, its curvature tensor R satisfies $R(Z, W) = 0$ as long as Z, W are (1, 0) complex vector fields on M.

This work is aimed at proving a Cartan-Norden-like theorem for affine Kähler immersions, generalizing the classical result in affine differential geometry (see [N-Pi]). In §1 we deal with some preliminaries about affine Kähler immersions in order to make our work self-contained. In §2 we prove our main result: if a non-flat Kähler manifold (M^n, g) can be affine Kähler immersed into $\mathbf{C}^{n+1}$ and the immersion f is non-degenerate, then for every point $x \in M^n$ we can find a parallel pseudokählerian metric in $\mathbf{C}^{n+1}$ such that f is locally isometric around the point x.

§ 1. Preliminaries

Throughout this work we shall refer to [N-Pi-Po] for basic results in the geometry of affine Kähler immersions. We recall here some fundamental equations. Let M^n be an n-dimensional complex manifold with complex structure J and let f: $M^n \to \mathbf{C}^{n+1}$ be a holomorhic immersion. We denote by D the standard flat connection in $\mathbf{C}^{n+1}$, a transversal (1, 0) vector field $\zeta = \xi - iJ\xi$ along f is said to be antiholomorphic if $D_Z\zeta = 0$ for every complex vector field Z of type (1, 0) on M^n.

If X and Y are real vector fields on M^n, we can write

$$D_X(f_*Y) = f_*(\nabla_X Y) + h(X, Y)\xi + k(X, Y)J\xi \tag{1.1}$$

thus defining a torsionfree affine connection ∇ and symmetric tensors h

Received March 6, 1990.