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Nagoya Math. J.
Vol. 121 (1991), 97-125

THE HECKE ALGEBRA ON THE COHOMOLOGY OF $\Gamma_0(p_0)$

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§ 1. Introduction

Let p_0 be a prime, $p_0 > 3$ and $\Gamma_0(p_0)$, $\Gamma_1(p_0)$, as usual, the congruence subgroups of $\Gamma = PSL_2(\mathbb{Z})$.

$$\Gamma_0(p_0) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \,\middle|\, c \equiv 0 \bmod p_0\right\},$$
$$\Gamma_1(p_0) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(p_0) \,\middle|\, d \equiv 1 \bmod p_0\right\}.$$

Denote

$$\Delta = \left\{r = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, a, b, c, d \in \mathbb{Z},\ \gcd(a, b, c, d) = 1,\ \det(r) \not\equiv 0 \bmod p_0\right\},$$
$$\Delta_0 = \left\{r = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Delta \,\middle|\, c \equiv 0 \bmod p_0\right\},$$
$$\Delta_1 = \left\{r = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Delta_0 \,\middle|\, d \equiv 1 \bmod p_0\right\}$$

with $\Delta_1 \subset \Delta_0 \subset \Delta$ and $\Delta_0/\Delta_1 \cong (\mathbb{Z}/p_0)^*$. Let $R = \mathbb{Z}[\frac{1}{6}]$. We consider the following R-module $M_n = \{\sum_{v=0}^{n} a_v x^v y^{n-v} \mid a_v \in R\}$. The semigroup Δ acts on M_n via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} x^v y^{n-v} = (ax + cy)^v (bx + dy)^{n-v}.$$

Let η: $\Gamma_0(p_0)/\Gamma_1(p_0) \cong (\mathbb{Z}/p_0)^* \to R^*$ be the Legendre-symbol. We extend η to Δ_0 such that η acts trivially on Δ_1, i.e. η is a character from Δ_0/Δ_1 to R^*. Denote by R_η the R-module of rank 1 with a Δ_0-operation given by $s_0.1 = \eta(s_0)\cdot 1$, $\forall s_0 \in \Delta_0$. Set $M_{n,\eta} = M_n \otimes R_\eta$. This is then a $R[\Delta_0]$-module. The goal of the present paper is to investigate the Hecke algebra on the cohomology group $H^*(\Gamma_0(p_0), M_{n,\eta})$. Let $S_k(\Gamma_0(p_0), \eta)$, as usual, be the

Received February 26, 1990.