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## THE HECKE ALGEBRA ON THE COHOMOLOGY OF $\varGamma_{0}(p_{0})$

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## §1. Introduction

Let  $p_0$  be a prime,  $p_0 > 3$  and  $\Gamma_0(p_0)$ ,  $\Gamma_1(p_0)$ , as usual, the congruence subgroups of  $\Gamma = PSL_2(\mathbb{Z})$ .

$$egin{aligned} & \Gamma_{\mathfrak{g}}(p_{\mathfrak{g}}) = \left\{ egin{pmatrix} a & b \ c & d \end{pmatrix} \in \Gamma \, \Big| \, c \equiv 0 egin{pmatrix} & 0 egin{pmatrix} & 0 egin{pmatrix} & p_{\mathfrak{g}} \end{bmatrix}, \ & \Gamma_{\mathfrak{g}}(p_{\mathfrak{g}}) = \left\{ egin{pmatrix} a & b \ c & d \end{pmatrix} \in \Gamma_{\mathfrak{g}}(p_{\mathfrak{g}}) \, \Big| \, d \equiv 1 egin{pmatrix} & 1 egin{pmatrix} & 0 egin{pmatrix} & p_{\mathfrak{g}} \end{bmatrix}. \end{aligned} 
ight\}. \end{aligned}$$

Denote

$$egin{aligned} & arLambda &= \left\{ r = egin{pmatrix} a & b \ c & d \end{pmatrix} \Big| a, b, c, d \in \mathbb{Z}, \ ext{gcd} \left( a, b, c, d 
ight) &= 1, \ ext{det} \left( r 
ight) 
otin 0 \ ext{mod} \ p_0 
ight\}, \ & \mathcal{A}_0 &= \left\{ r = egin{pmatrix} a & b \ c & d \end{pmatrix} 
otin \mathcal{L} \left| c \equiv 0 \ ext{mod} \ p_0 
ight\}, \ & \mathcal{A}_1 &= \left\{ r = egin{pmatrix} a & b \ c & d \end{pmatrix} 
otin \mathcal{L} \left| d \equiv 1 \ ext{mod} \ p_0 
ight\}, \end{aligned}$$

with  $\Delta_1 \subset \Delta_0 \subset \Delta$  and  $\Delta_0/\Delta_1 \cong (\mathbb{Z}/p_0)^*$ . Let  $R = \mathbb{Z}[\frac{1}{6}]$ . We consider the following *R*-module  $M_n = \{\sum_{v=0}^n a_v x^v y^{n-v} | a_v \in R\}$ . The semigroup  $\Delta$  acts on  $M_n$  via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} x^{v} y^{n-v} = (ax + cy)^{v} (bx + dy)^{n-v}.$$

Let  $\eta: \Gamma_0(p_0)/\Gamma_1(p_0) \cong (\mathbb{Z}/p_0)^* \to R^*$  be the Legendre-symbol. We extend  $\eta$  to  $\mathcal{A}_0$  such that  $\eta$  acts trivially on  $\mathcal{A}_1$ , i.e.  $\eta$  is a character from  $\mathcal{A}_0/\mathcal{A}_1$  to  $R^*$ . Denote by  $R_\eta$  the R-module of rank 1 with a  $\mathcal{A}_0$ -operation given by  $s_0.1 = \eta(s_0) \cdot 1, \ \forall s_0 \in \mathcal{A}_0$ . Set  $M_{n,\eta} = M_n \otimes R_\eta$ . This is then a  $R[\mathcal{A}_0]$ -module. The goal of the present paper is to investigate the Hecke algebra on the cohomology group  $H^*(\Gamma_0(p_0), M_{n,\eta})$ . Let  $S_k(\Gamma_0(p_0), \eta)$ , as usual, be the

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