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YOUNG DIAGRAMMATIC METHODS IN NON-COMMUTATIVE INVARIANT THEORY

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Introduction

In this paper we will study some aspects of non-commutative invariant theory. Let V be a finite-dimensional vector space over a field K of characteristic zero and let

$$K[V] = K \oplus V \oplus S^2(V) \oplus \cdots, \text{ and}$$
$$K\langle V\rangle = K \oplus V \oplus \otimes^2 V \oplus \otimes^3 V \oplus \cdots$$

be respectively the symmetric algebra and the tensor algebra over V. Let G be a subgroup of $GL(V)$. Then G acts on $K[V]$ and $K\langle V\rangle$. Much of this paper is devoted to the study of the (non-commutative) invariant ring $K\langle V\rangle^G$ of G acting on $K\langle V\rangle$.

In the first part of this paper, we shall study the invariant ring in the following situation.

Take a classical group G (i.e., $G = SL(n, K)$, $O(n, K)$ or $Sp(n, K)$) and the standard G-module K^n. Let V be the d-th symmetric power of K^n. Then G acts on V and we get $K\langle V\rangle^G$.

By the Lane-Kharchenko theorem ([L], [Kh]), the invariant ring $K\langle V\rangle^G$ is a free algebra. For the construction of explicit free generators, we will develop a symbolic method along the lines of Kung-Rota [K-R].

In the second part of this paper, we will study S-algebras in the sence of A.N. Koryukin. Koryukin [Ko] has proved that if V is a finite-dimensional K-vector space and G is a reductive subgroup of $GL(V)$ then $K\langle V\rangle^G$ is finitely generated as an S-algebra. We will prove that a homogeneous system of generators for the (commutative) invariant ring $K[\Lambda^2 V \oplus V]^G$ gives rise to a system of generators for the invariant ring $K\langle V\rangle^G$ as an S-algebra.

In the final part of this paper, we will study (non-commutative) in-

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