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SELF-LINKED CURVE SINGULARITIES

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Introduction

Let S be a three-dimensional regular local ring and let I be a prime ideal in S of height two. This paper is motivated by the question of when I is a set-theoretic complete intersection and when the symbolic Rees algebra $S(I) = \bigoplus_{n\geq 0} I^{(n)}t^n$ is Noetherian. The connection between the two problems is given by a result of Cowsik which says that the Noetherian property of $S(I)$ implies that I is a set-theoretic complete intersection ([1]).

The ideal I is said to be *linked* to an S-ideal J if there exists an S-regular sequence α_1, α_2 in $I \cap J$ such that $J = (\alpha_1, \alpha_2)\colon I$ and $I = (\alpha_1, \alpha_2)\colon J$, and I is called *self-linked* if I is linked to I ([14]) (see also [15], [21], [22], where such ideals were studied). Of course every self-linked ideal is a set-theoretic complete intersection because $I^2 \subset (\alpha_1, \alpha_2)$. As one of the main results in the first section of this paper, we prove that I is self-linked in case S/I has multiplicity at most five (Corollary 1.14). This follows from another result that gives a criterion in terms of the resolution, for when an almost complete intersection is self-linked (Theorem 1.1 and Proposition 1.8) (parts of the criterion are similar to results of Szpiro ([20]), Ferrand, Valla ([22]), Mohan Kumar). Using this criterion, we also characterize all self-linked monomial space curves (Corollary 1.10) and we show that normal almost complete intersections are self-linked (Corollary 1.9) (an ideal is called normal if all its powers are integrally closed). As an immediate consequence of Theorem 1.8, we also obtain Kumar's result that an ideal linked to a regular ideal is self-linked (Corollary 1.11).

In the second section we study the question of when $S(I) = S[It, I^{(2)}t^2]$. Of course this equality forces $S(I)$ to be Noetherian. Here we will always assume that I is an almost complete intersection in S and that the ideal

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