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QUANTIZATION OF A POISSON ALGEBRA AND POLYNOMIALS ASSOCIATED TO LINKS

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Dedicated to Professor Masahisa Adachi on the occasion of his 60th birthday

§ 0. Introduction

A formal quantization of Poisson algebras was discussed by several authors (see for instance Drinfel'd [D]). A formal Lie algebra generated by *homotopy classes of loops* on a Riemann surface Σ was obtained by W. Goldman in [G], and its Poisson algebra was quantized, in the sense of Drinfel'd, by Turaev in [T]. Briefly speaking, *a quantization of a commutative Poisson algebra P* is a noncommutatively extented algebra A such that the noncommutativity is related to the Lie bracket of the Poisson algebra P through a surjective homomorphism ρ: $A \to P$ (see Definition 2.1). In Turaev's quantization, the algebra A is a semi-group algebra generated by *links* in the thickened Riemann surface ((Riemann surface Σ) $\times$ (real line $\mathbf{R}$)).

On the other hand, there is an obvious map from Goldman's Poisson algebra to the ring $\mathbf{Z}[H_1]$ of polynomials with integral coefficients on the first homology group H_1 of the Riemann surface which assigns to each loop its homology class. There arises a natural question whether it is possible to construct a map from Turaev's algebra to the ring $\mathbf{Z}[H_1][h]$, the polynomial ring with coefficient in $\mathbf{Z}[H_1]$, such that the diagram

$$\begin{array}{ccc} A & \longrightarrow & \mathbf{Z}[H_1][h] \\ {\scriptstyle \text{quantization}}\downarrow & & \downarrow{\scriptstyle h \to 0} \\ P & \longrightarrow & \mathbf{Z}[H_1] \end{array}$$

commutes. Our main purpose in this paper is to show that the *polynomial invariant of links* introduced in § 3 (Definition 3.1) gives an answer

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