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ON FANO MANIFOLDS, WHICH ARE $\boldsymbol{P}^K$-BUNDLES OVER $\boldsymbol{P}^2$

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In our earlier paper [8] we discussed Fano manifolds X that are of the form $X = \mathbb{P}(\mathscr{E})$ with a rank-2 vector bundle $\mathscr{E}$ on a surface S. Here we study a more general situation of Fano manifolds, ruled over the complex projective plane $\boldsymbol{P}^2$ as $\boldsymbol{P}^{r-1}$-bundles, i.e., being of the form $\mathbb{P}(\mathscr{E})$ with $\mathscr{E}$-a bundle of rank $r \geq 3$ on $\boldsymbol{P}^2$. We say that $\mathscr{E}$ is a Fano bundle if $\mathbb{P}(\mathscr{E})$ is a Fano manifold.

THEOREM. *All manifolds $X = \mathbb{P}(\mathscr{E})$ with $\mathscr{E}$ a vector bundle of rank $r \geq 2$ over $\boldsymbol{P}^2$ are listed below*:

c_1	c_2	$\mathscr{E}$	$X = \mathbb{P}(\mathscr{E})$
0	0	$\mathscr{O}^r$	$\boldsymbol{P}^2 \times \boldsymbol{P}^{r-1}$
1	0	$\mathscr{O}(1) \oplus \mathscr{O}^{r-1}$	*blow-up of* $\boldsymbol{P}^{r+1}$ *along* $\boldsymbol{P}^{r-1}$
1	1	$\boldsymbol{T}(-1) \oplus \mathscr{O}^{r-2}$	*a divisor of degree* (1, 1) *in* $\boldsymbol{P}^2 \times \boldsymbol{P}^r$
2	0	$\mathscr{O}(2) \oplus^{r-1}$	*blow-up of the cone in* $\boldsymbol{P}^{r+4}$ *over veronese* $(\boldsymbol{P}^2) \subset \boldsymbol{P}^5$ *along its vertex* $(= \boldsymbol{P}^{r-2})$
2	1	$\mathscr{O}(1)^2 \oplus \mathscr{O}^{r-2}$ $(r \geq 3)$	*blow-up of the cone in* $\boldsymbol{P}^{r+3}$ *over* $\boldsymbol{P}^1 \times \boldsymbol{P}^2 \subset \boldsymbol{P}^5$ *along its vertex* $(= \boldsymbol{P}^{r-3})$
2	2	$r = 2$: *a bundle* $\mathscr{E}_2$ *in* $0 \to \mathscr{O} \to \mathscr{E}_2(-1) \to \boldsymbol{J}_x \to 0$	*blow-up of* $\boldsymbol{Q}_3$ *along a line*;
		$r \geq 3$: *special*: $\mathscr{O}^{r-1} \oplus \mathscr{E}_2$ *general*: $\boldsymbol{T}(-1) \oplus \mathscr{O}(1) \oplus \mathscr{O}^{r-3}$	*blow-up of a cone over a smooth quadric* $\boldsymbol{Q}_3$ *(resp.* $\boldsymbol{Q}_4$*) along a linear subspace* $\boldsymbol{P}^{r-1}$ *containing the vertex* $\cong \boldsymbol{P}^{r-3}$ *(resp.* $\boldsymbol{P}^{r-4}$*)*
2	3	*All bundles fitting into* $0 \to \mathscr{O} \to (-1)^2 \to \mathscr{O}^{r+2} \to \mathscr{E} \to 0$	*blow-up of* $\boldsymbol{P}^3$ *along a twisted cubic curve, if* $r = 2$
		$r = 3$: *special* $\mathscr{E}_3^s = \mathscr{O} \oplus \mathscr{E}_2$	*blow-up of the cone over a twisted cubic curve in* $\boldsymbol{P}^4$, *i.e. the rational normal scroll* $\boldsymbol{S}(0, 3)$

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