N. Tu Cuong
Nagoya Math. J.
Vol. 120 (1990), 77-88

ON THE LENGTH OF THE POWERS OF SYSTEMS OF PARAMETERS IN LOCAL RING

NGUYEN TU CUONG

§ 1. Introduction

Throughout this note, A denotes a commutative local Noetherian ring with maximal ideal $\mathfrak{m}$ and M a finitely generated A-module with $\dim(M) = d$. Let $x_1, \cdots, x_d$ be a system of parameters (s.o.p. for short) for M and I the ideal of A generated by $x_1, \cdots, x_d$. We consider the length $l(M/(x_1^{n_1}, \cdots, x_d^{n_d})M)$ over A as a function in the positive integers $n_1, \cdots, n_d$. J-L. Garcia Roig and D. Kirby [5] have shown that this function is generally not a polynomial for $n_1, \cdots, n_d \gg 0$ (sufficiently large) but, if M is a generalized Cohen-Macaulay module, then

$$l(M/(x_1^{n_1}, \cdots, x_d^{n_d})M) = n_1 \cdots n_d e(I; M) + \sum_{i=0}^{d-1} \binom{d-1}{i} l(H_{\mathfrak{m}}^i(M))$$

for $n_1, \cdots, n_d \gg 0$, where $e(I; M)$ denotes the multiplicity of M relative to I and $H_{\mathfrak{m}}^i(M)$ is the i-th local cohomology module of M with respect to $\mathfrak{m}$. Therefore, it is natural to ask under which conditions $l(M/(x_1^{n_1}, \cdots, x_d^{n_d})M)$ is a polynomial for $n_1, \cdots, n_d \gg 0$? (see [9], Question 1.1).

The purpose of this note is to give an answer to this question. Before stating the main result we need the following definition. Let $x_1, \cdots, x_d$ be a s.o.p. for M. We say that $x_1, \cdots, x_d$ is a p-system of parameters (p-s.o.p. for short) for M if there exists a positive integer n_0 such that

$$(x_1^{n_1}, \cdots, x_{i-1}^{n_{i-1}})M : x_i^{n_i} = (x_1^{n_1}, \cdots, x_{i-1}^{n_{i-1}})M : x_i^{n_0}$$

for all $n_1, \cdots, n_d \geq n_0$, $i = 1, \cdots, d$ $(x_0 = 0)$.

We say that $x_1, \cdots, x_d$ is an unconditioned p-s.o.p. if for every permutation of the sequence $x_1, \cdots, x_d$, the above condition holds with respect to the same integer n_0.

THEOREM 1. *The function $l(M/(x_1^{n_1}, \cdots, x_d^{n_d})M)$ is a polynomial for*

Received July 21, 1989.