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INJECTIVE MODULES OVER TWISTED POLYNOMIAL RINGS

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Differential polynomial rings over a universal field and localized twisted polynomial rings over a separably closed field of non-zero characteristic twisted by the Frobenius endomorphism were the first domains not divisions rings that were shown to have every simple module injective (see [C] and [C-J]). By modifying the separably closed condition for the polynomial rings twisted by the Frobenius, the conditions of every simple being injective and only a single isomorphism class of simple modules were shown to be independent (see [O]). In this paper we continue the investigation of injective cyclic modules over twisted polynomial rings with coefficients in a commutative field.

Let κ be a field and σ an endomorphism of κ. We can then form the twisted polynomial ring $R = \kappa[X; \sigma]$ with

$$R = \left\{ \sum_{i=0}^{n} \alpha_i X^i \mid n \in \mathbf{Z},\ \alpha_i \in \kappa \right\}$$

under usual polynomial addition and multiplication given by the relation

$$X\alpha = \sigma(\alpha)X .$$

We are interested in non-zero cyclic injective left modules over this ring R.

It is well known (see [J]) that R is a left Euclidean domain using the degree function, and so a left principal ideal domain. Thus a left R-module is injective if and only if it is divisible (see [R, page 70]).

The field κ is an R-module under the action

$$\left(\sum_{i=0}^{n} p_i X^i \right) \cdot \alpha = \sum_{i=0}^{n} p_i \sigma^i(\alpha) .$$

Using this action, we get

THEOREM 1. *Let κ be a field and $R = \kappa[X; \sigma]$. Then the following are equivalent*:

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