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EVERY ALGEBRAIC KUMMER SURFACE IS THE K3-COVER OF AN ENRIQUES SURFACE

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Introduction

A *Kummer surface* is the minimal desingularization of the surface T/i, where T is a complex torus of dimension 2 and i the involution automorphism on T. T is an abelian surface if and only if its associated Kummer surface is algebraic. Kummer surfaces are among classical examples of K3-surfaces (which are simply-connected smooth surfaces with a nowhere-vanishing holomorphic 2-form), and play a crucial role in the theory of K3-surfaces. In a sense, all Kummer surfaces (resp. algebraic Kummer surfaces) form a 4 (resp. 3)-dimensional subset in the 20 (resp. 19)-dimensional family of K3-surfaces (resp. algebraic K3 surfaces).

An *Enriques surface* is a smooth projective surface Y with $2K_Y = 0$, $H^1(Y, \mathcal{O}_Y) = H^2(Y, \mathcal{O}_Y) = 0$. The unramified double cover of Y defined by the torsion class K_Y is an algebraic K3-surface. Conversely, if an algebraic K3-surface X admits a fixed-point-free involution τ, then the quotient surface X/τ is an Enriques surface. It is known that all Enriques surfaces form a 10-dimensional moduli space.

Let X be a surface. The *Picard number* of X, denoted by $\rho(X)$, is the rank of the *Néron-Severi group* $\mathrm{NS}(X)$, the sublattice of $H^2(X, \boldsymbol{Z})$ generated by algebraic cycles. The *transcendental lattice* T_X of X is the orthogonal complement of $\mathrm{NS}(X)$ in $H^2(X, \boldsymbol{Z})$. If X is a K3-surface, then $0 \leq \rho(X) \leq 20$. If X is the K3-cover of an Enriques surface, then $\rho(X) \geq 10$.

Let L be a lattice, i.e. a free $\boldsymbol{Z}$-module of finite rank together with a $\boldsymbol{Z}$-valued symmetric bilinear form. For every integer m we denote by $L(m)$ the lattice obtained from L by multiplying the values of its bilinear form by m. The *length* of L, denoted by $l(L)$, is the minimum number of generators of $L^*/j(L)$, where $j: L \to L^* = \mathrm{Hom}(L, \boldsymbol{Z})$ is the natural

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