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EISENSTEIN SERIES IN HYPERBOLIC 3-SPACE AND KRONECKER LIMIT FORMULA FOR BIQUADRATIC FIELD

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§ 0. Introduction

Let $L = kK$ be the composite of two imaginary quadratic fields k and K. Suppose that the discriminants of k and K are relatively prime. For any primitive ray class character χ of L, we shall compute $L(1, \chi)$ for the Hecke L-function in L. We write $\mathfrak{f}$ for the conductor of χ and C for the ray class modulo $\mathfrak{f}$. Let $\mathfrak{c} \in C$ be any integral ideal prime to $\mathfrak{f}$. We write $\mathfrak{a} = \mathfrak{c}/(\vartheta_L \mathfrak{f}) = \mathfrak{g}\omega_1 + \mathfrak{n}\omega_2$ as $\mathfrak{g}$-module where $\mathfrak{g}, \mathfrak{n}$ and ϑ_L are, respectively, the ring of integers in k, an ideal in k and the differente of L. Let $L(s, \chi) = T(\chi)^{-1} \sum_C \bar{\chi}(C)\Psi(C, s)$ where $T(\chi)$ is the Gaussian sum and, as in (3.2),

$$\Psi(C, s) = N_{L/Q}(\mathfrak{a})^s \sum_{(\mu)\mathfrak{f}}'' e^{2\pi i Tr_{L/Q}(\mu)} |N_{L/Q}(\mu)|^{-s} .$$

In § 1, 2, for each pair of ideals $(\mathfrak{m}, \mathfrak{n})$ in k, we associate Eisenstein series in hyperbolic 3-space having characters. For this series, we show the Kronecker limit formula. In § 3, 4, we show that $\Psi(C, s)$ is written as the constant term in the Fourier expansion of the Eisenstein series with reference to the hyperbolic substitution of $SL_2(k)$ (Theorems 4.3, 4.4). In § 5, we compute the Kronecker limit formula for $\Psi(C, s)$ (Theorems 5.6, 5.7). The limit formula is written as the Fourier cosine series of $\omega + \tilde{\omega}$ ($\omega = \omega_1^{-1}\omega_2$) whose coefficients are functions of $\omega - \tilde{\omega}$ where $\tilde{\omega}$ is the conjugate of ω over k.

NOTATIONS. We denote by $\boldsymbol{Z}$, $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{C}$, respectively, the ring of rational integers, the rational number field, the real number field and the complex number field. For $z \in \boldsymbol{C}$, $\bar{z}$ denotes the complex conjugate of z. We write $S(z) = z + \bar{z}$ and $|z|^2 = z\bar{z}$. For $z \in \boldsymbol{C}$, $\sqrt{z}$ means $-\pi/2 < \arg \sqrt{z} \leqq \pi/2$. For an associative ring A with identity element, $A^\times$

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