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POLYNOMIAL RINGS AND THEIR PROJECTIVE MODULES

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§ 1. Introduction

Let R be a regular noetherian ring. A central question concerning projective modules over polynomial R-algebras is the following.

(1.1) BASS-QUILLEN CONJECTURE ([2] Problem IX, [10]). *Every finitely generated projective module P over a polynomial R-algebra $R[T]$, $T = (T_1, \cdots, T_n)$ is extended from R, i.e.*

$$P \cong R[T] \otimes_R P/(T)P .$$

When $\dim R \leqslant 1$ the conjecture is true by the Quillen-Suslin Theorem (see [10], [13]). Also H. Lindel [5] proved that the conjecture is true for regular rings essentially of finite type over a field. His ideas work also in some mixed characteristic cases (cf. [7]; see also Proposition (2.1) and Theorem (4.1) below which we included here for the sake of completeness). These results show that it is worth to consider the following

(1.2) QUESTION ([7]). *A regular local ring is a filtered inductive limit of regular local rings essentially of finite type over Z.*

In [8] Corollary (2.7) we stated (1.2) in the equal characteristic case which gave us the possibility to solve in [9] (using [3]) a question of Quillen (see [10]) (and so the BQ Conjecture) in the equal characteristic case.

In this paper we obtain some results in the mixed characteristic case concerning these two questions. Our Theorem (3.1) says that (1.2) is true for a regular local ring $(A, \mathfrak{m})$ when either $p := \operatorname{char}(A/m) \notin \mathfrak{m}^2$, or A is excellent henselian. Using Lindel's results we solve the BQ Conjecture for regular noetherian rings R such that for every prime integer t, either t is an unit in R, or $R/t\,R$ is a regular ring (see Corollary (4.3)). If $p \in \mathfrak{m}^2$ our Corollary (2.4) and Theorem (3.1) iii) show that it is "almost" enough

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