FLOPS

JÁNOS KOLLÁR

§1. Introduction

The aim of this note is to study birational transformations of three-folds with nef canonical classes. One would like to write any such map as a composite of certain simple and basic transformations.

A special case was first considered by Kulikov [Ku] and later extensively studied by several authors. The next main conceptual step was Reid's study [R2] of small resolutions of terminal threefold singularities. He obtained the elementary birational transformations as follows: find a copy of such a small resolution inside the threefold and then replace it with another resolution. This approach was extended by Kawamata [K3] to canonical singularities.

If $g: X \dashrightarrow X'$ is a birational transformation between threefolds with nef canonical classes then using the above results one can start factoring g into the composite of such elementary transformations. I will call them flops. It is however not clear that the process will ever stop. The main contribution of this note is a simple proof of this fact. Besides extending the scope of applications it allows one to simplify considerably the proof of the existence of flops given in [K3].

Section two contains a proof of the existence of flops in the terminal case. This quick proof is based on an idea of Mori. I am grateful to him for allowing me to present it here. Section three is devoted to a special case of the termination of flops. In the next section this is used to solve the above mentioned factorisation problem of birational maps. Finally in sections five and six a simple proof of the main theorem of [K3] is given. The arguments show that going from terminal to canonical singularities is rather easy in all dimensions.

Most of the proofs work for algebraic and analytic threefolds as well.