R. A. Mollin and H. C. Williams
Nagoya Math. J.
Vol. 112 (1988), 143-151

ON PRIME VALUED POLYNOMIALS AND CLASS NUMBERS OF REAL QUADRATIC FIELDS

R.A. MOLLIN AND H.C. WILLIAMS

§1. Introduction

Gauss conjectured that there are infinitely many real quadratic fields with class number one. Today this is still an open problem. Moreover, as Dorian Goldfeld, one of the recipients of the 1987 Cole prize in number theory (for his work on another problem going back to Gauss) recently stated in his acceptance of the award: "This problem appears quite intractible at the moment." However there has recently been a search for conditions which are tantamount to class number one for real quadratic fields. This may be viewed as an effort to shift the focus of the problem in order to understand more clearly the inherent difficulties, and to reveal some other beautiful interrelationships.

One of the avenues of inquiry has been to search for criteria in terms of prime valued polynomials. The impetus for this search has come from a result for complex quadratic fields which is almost 75 years old.

THEOREM 1.1 (Rabinowitsch [13] and [14]). *If* $d \equiv 3 \pmod 4$ *where* d *is a positive square-free integer, then* $p(x) = x^2 - x + (d+1)/4$ *is prime for all integers* x *with* $1 \leq x \leq (d-3)/4$ *if and only if* $h(-d) = 1$.

It is now well-known (see [2], [5], and [17]) that there exactly nine complex quadratic fields with class number one. They are $Q(\sqrt{-d})$ for $d \in \{1, 2, 3, 7, 11, 19, 43, 67, 163\}$. Thus together with Theorem 1.1 we get the remarkable:

THEOREM 1.2. *If* $d \equiv 3 \pmod 4$ *is a square-free integer then* $x^2 - x + (d+1)/4$ *is prime for all integers* x *with* $1 \leq x \leq (d-3)/4$ *if and only if* $d \in \{7, 11, 19, 43, 67, 163\}$.

Given Gauss' open conjecture cited at the outset, the story for real

Received May 25, 1987.