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CALCULUS ON GAUSSIAN AND POISSON WHITE NOISES

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§ 1. Introduction

Recently one of the authors has introduced the concept of generalized Poisson functionals and discussed the differentiation, renormalization, stochastic integrals etc. ([8], [9]), analogously to the works of T. Hida ([3], [4], [5]). Here we introduce a transformation $\mathscr{S}_P$ for Poisson fnnctionals with the idea as in the case of Gaussian white noise (cf. [10], [11], [12], [13]). Then we can discuss the differentiation, renormalization, multiple Wiener integrals etc. in a way completely parallel with the Gaussian case. The only one exceptional point, which is most significant, is that the multiplications are described by

$$x^G(t)\cdot = \partial_t^* + \partial_t \qquad \text{for the Gaussian case,}$$
$$x^P(t)\cdot = (\partial_t^* + 1)(\partial_t + 1) \qquad \text{for the Poisson case,}$$

as will be stated in Section 5. Conversely, those formulae characterize the types of white noises.

In Section 2, we will define Gaussian and Poisson white noises on a general parameter space T, which is a separable topological space with a σ-finite non-atomic Borel measure ν. Let $\mathscr{E} \subset L^2(T, \nu) \subset \mathscr{E}^*$ be a Gel'fand triplet satisfying the assumptions [A.1], [A.2], [A.3] in Section 2. Then the measure of Gaussian white noise μ_G and the measure of Poisson white noise μ_P are characterized respectively by their Fourier transforms

$$\int_{\mathscr{E}^*} \exp[i\langle x, \xi\rangle] d\mu_G(x) = \exp\left[-\frac{1}{2}\int_T |\xi(t)|^2 d\nu(t)\right],$$
$$\int_{\mathscr{E}^*} \exp[i\langle x, \xi\rangle] d\mu_P(x) = \exp\left[\int_T (\exp[i\xi(t)] - 1) d\nu(t)\right],$$

with ξ in $\mathscr{E}$. Then we introduce transformations $\mathscr{S}_G$ from $L^2(\mathscr{E}^*, \mu_G)$ and $\mathscr{S}_P$ from $L^2(\mathscr{E}^*, \mu_P)$ to the same space $\mathscr{F}^{(0)}$ which is a Hilbert space with the reproducing kernel $\exp[\langle \xi, \eta\rangle]$, $\eta \in \mathscr{E}$.

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