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STOCHASTIC DIFFERENTIAL GAMES AND VISCOSITY SOLUTIONS OF ISAACS EQUATIONS

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§ 1. Introduction

Recently P.L. Lions has demonstrated the connection between the value function of stochastic optimal control and a viscosity solution of Hamilton-Jacobi-Bellman equation [cf. 10, 11, 12]. The purpose of this paper is to extend partially his results to stochastic differential games, where two players conflict each other. If the value function of stochatic differential game is smooth enough, then it satisfies a second order partial differential equation with max-min or min-max type nonlinearity, called Isaacs equation [cf. 5]. Since we can write a nonlinear function as min-max of appropriate affine functions, under some mild conditions, the stochastic differential game theory provides some convenient representation formulas for solutions of nonlinear partial differential equations [cf. 1, 2, 3].

Now we will consider stochastic differential games on a finite interval [0, T], for simplicity. Let Γ_i be a compact and convex subset of R^{k_i}. $B(t)$, $t \geq 0$, denotes a standard d-dimensional Brownian motion, defined on a probability space (Ω, F, P). A B-adapted process U is called a control of player i, if $U_i(t) \in \Gamma_i$. We denote the totality of controls of player i by A_i, equipped with $L_2([0, T] \times \Omega)$ − topology.

For $U_i \in A_i$, $i = 1, 2$, the system X is evoluted by the following controlled stochastic differential equation (CSDE in short),

$$(1.1)\qquad \begin{cases} dX(t) = \alpha(X(t), U_1(t), U_2(t))dB(t) + \Upsilon(X(t), U_1(t), U_2(t))dt \\ X(0) = \chi \end{cases}$$

where α and Υ are symmetric matrix and vector valued functions, defined on $R^d \times \Gamma_1 \times \Gamma_2$, respectively. We assume some regularity, see (A1) and (A2). The solution X of (1, 1) is denoted by $X(t, \chi, U_1, U_2)$, if χ, U_1 and U_2

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