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ON THE DIVISOR CLASS GROUPS OF A TWO-DIMENSIONAL LOCAL RING AND ITS FORM RING

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Introduction

Let A be a noetherian ring and let I be an ideal of A contained in the Jacobson radical of A: $\mathrm{Rad}(A)$. We assume that the form ring of A with respect to the ideal I: $G = \mathrm{Gr}(A, I)$, is a normal integral domain. Hence A is a normal integral domain and one can ask for the links between $\mathrm{Cl}(A)$ and $\mathrm{Cl}(G)$.

Let $R = \oplus_{n\in Z} I^n$ be the Rees algebra of A with respect to the ideal I (see § 2). In a previous paper [20], the authors have proved that $\mathrm{Cl}(A) \simeq \mathrm{Cl}(R)$; moreover there exists a "canonical" map $j\colon \mathrm{Cl}(R) \to \mathrm{Cl}(G)$ deduced from the hypersurface section $R \to G = R/uR$ (§ 1). Following the ideas of Lipman's paper [18], in [20] an attempt was made to find out sufficient conditions for $\ker(j) = 0$, (resp.: for $\ker(j)$ to be a torsion group). But this sufficient conditions become almost tautological when $\dim(A) = 2$ and $\mathrm{ht}(I) = 2$ (i.e. when A is a local ring and $I = \mathrm{Rad}(A)$; see §1). This paper deals with this last case.

The main result of the paper is Theorem 4; this theorem can be proved also by using the geometrical machinery of Grothendieck, Danilov, Boutot and Bădescu-Fiorentini [15, 8, 9, 6, 3] (see also the Remark 3 after the proof of Theorem 4).

Our proof mainly uses simple tools of Commutative Algebra and standard facts of Local Cohomology theory. A key point is the finiteness of a suitable local cohomology module which we derive from [15].

It is also interesting that the short exact sequences which appear in Theorem 1 of [18] are the same which appear in our proof. In a certain sense, this circumstance unifies the two techniques.

However the problem of the injectivity of $j\colon \mathrm{Cl}(R) \to \mathrm{Cl}(G)$ for a general hypersurface section $R \to G = R/uR$, $\dim(G) = 2$, is rather different

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