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MULTIPLICITY AND t-ISOMULTIPLE IDEALS

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Introduction

Let V be an irreducible non degenerate variety in P^n; a classical geometric result says that degree $(V) \geq \operatorname{codim} V + 1$ and, if equality holds, V is said to be of minimal degree. Varieties of minimal degree has been classified by Del Pezzo and Bertini and they all are intersections of quadrics. The local version of this result is due to J. Sally who proved that if $(A, \mathfrak{N})$ is a regular local ring and $(R = A/I,\ \mathfrak{M} = \mathfrak{N}/I)$ is a Cohen-Macaulay local ring of minimal multiplicity, according to the bound $e(R) \geq \operatorname{height}(I) + 1$ given by Abhyankar, then the tangent cone $\operatorname{gr}_{\mathfrak{M}}(R)$ of R is intersection of quadrics and it is Cohen-Macaulay.

On the other hand if $I \subset \mathfrak{N}^2$ and $S_R(\mathfrak{M})$ is the symmetric algebra of the R-module $\mathfrak{M}$, then by a result of A. Micali we know that $S_R(\mathfrak{M})$ is not a domain; however J. Risler proved that, if R is reduced, then $S_R(\mathfrak{M})$ is reduced if and only if $\operatorname{gr}_{\mathfrak{M}}(R)$ is intersection of quadrics.

Recently J. Elias considered the case I is a perfect codimension 2 ideal of the regular local ring $(A, \mathfrak{N})$; if $v = v(I)$ is the minimal number of generators of I, he proved that $e(A/I) \geq \binom{v}{2}$ and, if equality holds, $\operatorname{gr}_{\mathfrak{M}}(R)$ is intersection of hypersurfaces of degree $v - 1$.

Further if one tries to extend the theory of normal flatness along permissible ideals to the non regular case, then it is natural to consider ideals whose corresponding tangent cone is intersection of hypersurfaces of the same degree t (see [Br]).

We say that an ideal I is t-isomultiple if $\operatorname{gr}_{\mathfrak{M}}(R)$ is defined by equations of the same degree t; this means that I has a standard base of elements of order t. As it turns out by the preceding examples, very often ideals with "minimal" multiplicity are t-isomultiple. In this paper we pursue this line in order to identify some interesting classes of t-isomultiple ideals.

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